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# RESEARCH MEMORANDUM

THEORETICAL ANALYSES TO DETERMINE UNBALANCED  
TRAILING-EDGE CONTROLS HAVING MINIMUM  
HINGE MOMENTS DUE TO DEFLECTION  
AT SUPERSONIC SPEEDS

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**NATIONAL ADVISORY COMMITTEE  
FOR AERONAUTICS**

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## NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

## RESEARCH MEMORANDUM

## THEORETICAL ANALYSES TO DETERMINE UNBALANCED

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## SUMMARY

Analyses based on theoretical results of NACA Report 1041 have been made to determine the plan forms of unbalanced trailing-edge flap-type controls having minimum hinge moments due to deflection and requiring minimum work to overcome the hinge moments due to deflection at supersonic speeds. Ratios of lift and rolling moment to hinge moment and ratios of lift and rolling moment to deflection work at fixed values of lift and rolling effectiveness were used as bases for the analyses.

Results of the analyses for longitudinal controls show high-aspect-ratio untapered controls to possess maximum ratios of lift  $L$  to hinge moment  $H$ . When low-aspect-ratio controls must be used, however, controls with triangular plan forms and highly swept hinge lines are shown to have higher values of  $L/H$  than untapered controls. Ratios of lift to deflection work for untapered controls are in most cases shown to be higher than those for controls with tapered plan forms.

On wings with sweptforward and unswept trailing edges, inversely tapered controls with triangular plan forms of moderate or low aspect ratio are shown to have maximum ratios of rolling moment  $L'$  to hinge moment  $H$ . On wings with sweptback trailing edges, maximum values of  $L'/H$  are shown for either untapered or normally tapered controls.

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For any given control shape, the analysis illustrates the importance of using small controls with high deflections to obtain large values of  $L'/H$ .

Maximum ratios of rolling moment to deflection work on wings with sweptforward trailing edges are in most cases obtained with inversely tapered controls with triangular plan forms. On wings with unswept and sweptback trailing edges, the deflection work required is near minimum for untapered controls with spans of about two-thirds of the wing semispan. Results indicate that large controls will in most cases have higher ratios of rolling moment to deflection work than smaller controls.

### INTRODUCTION

The control forces on aircraft operating at supersonic speeds are so high that very substantial power boost systems are usually required to handle the hinge moments. As an approach to a solution to the problem of reducing the size and work requirements of boost systems for such aircraft, theoretical analyses have been made of the hinge moments due to deflection of unbalanced trailing-edge flap-type controls with plan forms varying throughout the range in which the control leading and trailing edges are supersonic and the control tips are streamwise. Aeroelastic effects were not included and the analyses, which are based on equations and charts from reference 1, are subject to the limitations of linearized theory.

In making the analyses, values of lift and rolling moment coefficients and parameters indicative of ratios of lift and rolling moment to hinge moment ( $F_L$  and  $F_l$ ) were calculated for a range of control plan forms on wings having various trailing edge sweep angles. (Trailing edge sweep angle was the only wing plan form parameter which had to be specified because the loading over the portion of a wing ahead of a control is not influenced by control deflection at supersonic speeds.) From these calculations, families of controls having fixed amounts of effectiveness were determined and the corresponding parameters,  $F_L$  and  $F_l$ , were plotted as functions of the various control plan-form parameters. From the resulting charts, the plan forms of controls producing fixed amounts of lift and rolling moment with minimum hinge moments due to deflection were determined. Similar analyses were also made to determine the plan forms of controls requiring minimum amounts

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of work to overcome the hinge moments due to deflection. (The analyses for deflection work are similar to analyses carried out by Jones and Cohen for the incompressible case and presented in ref. 2.) The hinge moment analyses will be applicable in cases where the strength of the actuating mechanism or the amount of torque available at the control are the design criteria. The work analyses will be applicable when the design criteria are the energy which must be carried for operating the boost system or the energy which the pilot must exert in event of boost failure.

Hinge moments due to angle of attack, damping in pitch and rolling depend on wing plan form and to varying degrees on complete aircraft configuration and have not been included in the present analyses because the calculations involved would have been exorbitant. The following comments regarding these neglected hinge moments should therefore be kept in mind in applying results of the analyses: Hinge moments due to angle of attack and damping in pitch are of primary importance with regard to longitudinal controls because hinge moments of such controls are equal to the algebraic summation of these hinge moments and hinge moments due to deflection. Consequently, complete analyses for longitudinal controls would require, in addition to the analyses of the present paper, similar analyses in which hinge moments due to angle of attack and damping in pitch are considered. With regard to the combined hinge moments of differentially deflected lateral controls, hinge moments due to angle of attack and damping in pitch are of no significance because the effects on opposite ailerons cancel. It is possible, however, that in some cases, the hinge moments of the individual ailerons will be of more importance than their combined hinge moments; for instance, when the ailerons are not interconnected but are actuated independently. In such cases, hinge moments due to angle of attack and damping in pitch would have to be considered. Hinge moments due to rolling are of primary importance with regard to lateral controls because in most cases they tend to reduce the hinge moments due to deflection of ailerons on both wing panels. It is estimated for the unbalanced trailing-edge-flap type of controls considered in the present paper that hinge moments due to rolling in the most critical cases are not likely to reduce hinge moments due to deflection by more than 15 or 20 percent. Hinge moments of this order are certainly of importance with regard to the actuation of controls but are probably of minor importance with regard to the selection of low hinge-moment controls. The hinge moments of longitudinal controls due to rolling are probably of less significance because the controls are usually located considerably nearer the axis of symmetry than lateral controls and consequently in regions where the induced angles of attack due to rolling are smaller.

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## SYMBOLS

M	free-stream Mach number
$\beta = \sqrt{M^2 - 1}$	
q	free-stream dynamic pressure
$\Lambda$	angle of sweep of wing leading edge, positive when sweptback
$\Lambda_{HL}$	angle of sweep of control hinge line, positive when sweptback
$\Lambda_{TE}$	angle of sweep of wing trailing edge, positive when sweptback
$\delta$	angle of control-surface deflection measured in streamwise direction, degrees
$\delta_1$	maximum value of $\delta$
$a = \frac{\tan \Lambda_{HL}}{\beta}$	
b	wing span
A	wing aspect ratio
$\lambda$	wing taper ratio
S	wing area
$y_f$	distance from wing-root chord to inboard parting line of control
$b_f$	control span
$c_t$	tip chord of control
$c_r$	root chord of control
$\lambda_f$	control-surface taper ratio ( $c_t/c_r$ )
$S_f$	area of control surface
$A_f$	aspect ratio of control surface ( $b_f^2/S_f$ )

$M_a$  area moment of control about hinge axis  
 $L$  lift induced by control deflection  
 $L'$  moment about wing-root chord induced by control deflection  
 $H$  hinge moment due to control deflection  
 $H_{(p)}$  hinge moment due to rolling  
 $W$  work required to overcome hinge moments due to control deflection (deflection work)

$$F_L = \beta b \cos \Lambda_{HL} \frac{L}{H} = \frac{\delta_1}{57.3} \frac{\beta b}{2} \frac{L}{W}$$

$$F_L' = \beta \cos \Lambda_{HL} \frac{L'}{H} = \frac{\delta_1}{57.3} \frac{\beta}{2} \frac{L'}{W}$$

$$C_L = \frac{L}{qS}$$

$$C_L' = \frac{L'}{qbS}$$

$$C_h = \frac{H}{2M_a q}$$

$p$  rolling rate (radians/sec)

$V$  velocity (feet/sec)

$\frac{pb}{2V}$  wing-tip helix angle

$$C_{L_p} = \frac{\partial C_L}{\partial \frac{pb}{2V}}$$

Subscript:

$\delta$  denotes partial derivative of force and moment coefficients with respect to  $\delta$

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## ANALYSIS

### General

The regions in which loading is influenced by the deflection of trailing-edge controls at supersonic speeds are limited to the surfaces of the deflected controls and to portions of the wings adjacent to the controls and lying within the Mach cones from the control tips (fig. 1). Within the scope of the present paper, the loadings induced by deflected controls are unaffected by wing plan form; and the wing trailing-edge sweep angle which defines the regions of induced loading on the wing is, therefore, the only wing parameter necessary for determining the characteristics of trailing-edge controls. It was convenient to choose various

values of  $\frac{\tan \Lambda_{TE}}{\beta}$  and for each of these values to vary systematically control plan form and location. The analyses, with fixed Mach numbers assumed, then correspond to examinations of the effects of control plan form and location on the characteristics of control surfaces on wings having various trailing-edge sweep angles.

All calculations were made for controls located on left wing panels so that positive (downward) control deflections would result in positive rolling moments as well as positive lift. Since positive control deflections in all cases result in negative hinge moments, the ratios of lift to hinge moment and rolling moment to hinge moment are negative and the functions of these ratios,  $F_L$  and  $F_l$ , presented in the charts, are also negative. These ratios and functions are discussed throughout the report in terms of absolute magnitude; that is, the most negative values are referred to as maximum.

The parameters used as bases for comparison in the analyses for determining maximum ratios of lift and rolling moment to hinge moment were  $F_L = \beta b \cos \Lambda_{HL} \left( \frac{L}{H} \right)$  and  $F_l = \beta \cos \Lambda_{HL} \left( \frac{L'}{H} \right)$ . The  $\beta$  and  $\cos \Lambda_{HL}$  terms were included in the parameters  $F_L$  and  $F_l$  in order to avoid considering Mach number as an independent variable in the calculations. When these parameters are used, Mach number enters the calculations only as part of the plan-form parameters  $\frac{\tan \Lambda_{HL}}{\beta}$ ,  $\frac{\tan \Lambda_{TE}}{\beta}$ , and  $\beta A_F$  (for untapered plan forms); consequently, for any given control plan form, variations in Mach number correspond to variations in  $\frac{\tan \Lambda_{HL}}{\beta}$ ,  $\frac{\tan \Lambda_{TE}}{\beta}$ , and/or  $\beta A_F$ . A linear dimension was needed to make the parameter  $F_L$  nondimensional and the  $b$  term was included in  $F_L$  for this purpose.



In using the parameters  $F_L$  and  $F_l$  as bases for comparison in determining the plan forms and locations of controls having maximum ratios of lift and rolling moment to hinge moment, the effects of the  $\cos \Lambda_{HL}$  term in the parameters  $F_L$  and  $F_l$  must be considered. Because of the  $\cos \Lambda_{HL}$  term, it is possible that values of  $F_L$  (for instance) for one value of  $\Lambda_{HL}$  may be higher than values for some higher absolute value of  $\Lambda_{HL}$ , whereas the value of  $L/H$  for the lower value of  $\Lambda_{HL}$  is less. This can be illustrated as follows:

$$\left(\frac{L}{H}\right)_2 = \frac{(\beta b \cos \Lambda_{HL})_1^{F_{L2}} \left(\frac{L}{H}\right)_1}{(\beta b \cos \Lambda_{HL})_2^{F_{L1}}} \quad (1)$$

(The subscript 1 refers to conditions corresponding to some arbitrary value of  $\Lambda_{HL}$  and subscript 2 refers to some other arbitrary value of  $\Lambda_{HL}$ ). In order to make proper comparisons of various control plan forms, it is necessary to assume a fixed Mach number and a fixed wing plan form. In such cases values of  $\beta$  and  $b$  will be constant and equation (1) reduces to:

$$\left(\frac{L}{H}\right)_2 = \frac{\cos \Lambda_{HL1}^{F_{L2}} \left(\frac{L}{H}\right)_1}{\cos \Lambda_{HL2}^{F_{L1}}} \quad (1a)$$

It can be seen that although  $F_{L2}$  may be less than  $F_{L1}$ , it is possible, depending on the ratios of the functions and the  $\cos \Lambda_{HL}$  terms, for  $\left(\frac{L}{H}\right)_2$  to be greater than  $\left(\frac{L}{H}\right)_1$ . Since the hinge line sweep parameter used in defining control plan form in the present paper is  $a = \frac{\tan \Lambda_{HL}}{\beta}$ , it is convenient for future reference to rewrite equation (1a) in terms of  $a$ :

$$\left(\frac{L}{H}\right)_2 = \left(\frac{L}{H}\right)_1 \frac{F_{L2}}{F_{L1}} \sqrt{\frac{1 + \beta^2 a_2^2}{1 + \beta^2 a_1^2}} \quad (1b)$$

The parameters  $F_L$  and  $F_l$  are also convenient for use in the analyses of controls on the basis of minimum deflection work. This can

be shown as follows:

$$W = \int_0^{s_1} F \, ds \quad (2)$$

where

$F$  force on control,  $\frac{H}{\bar{x}} = \frac{2M_a q \delta \bar{c} h \delta}{\bar{x}}$

$s$  deflection of point on control at which center of loading lies,  $\frac{\bar{x} \delta}{57.3 \cos \Lambda_{HL}}$

$\bar{x}$  distance from hinge axis to center of loading on control, measured normal to hinge axis

and subscript 1 denotes maximum displacement.

Rewriting equation (2)

$$W = \frac{2M_a q \bar{c} h \delta}{57.3 \cos \Lambda_{HL}} \int_0^{\delta_1} \delta \, d\delta \quad (2a)$$

Integrating and reducing:

$$W = \frac{\delta_1 H_1}{57.3 \times 2 \cos \Lambda_{HL}} \quad (2b)$$

Rearranging equation (2b)

$$H_1 = \frac{2 \cos \Lambda_{HL}}{\delta_1} W \quad (3)$$

When the value of hinge moment  $H_1$  from equation (3) is substituted in equations  $F_L = \beta b \cos \Lambda_{HL} \left( \frac{L}{H} \right)$  and  $F_z = \beta \cos \Lambda_{HL} \left( \frac{L'}{H} \right)$ , the parameters  $F_L$  and  $F_z$ , in terms of deflection work become:  $F_L = \frac{\delta_1}{57.3} \frac{\beta b}{2} \frac{L}{W}$  and

$F_L = \frac{\delta_1}{57.3} \frac{\beta}{2} \frac{L'}{W}$ . From these definitions it can be seen that in cases where comparisons are made of controls at equal deflections, maximum values of  $F_L$  and  $F_L'$  will correspond to maximum values of  $L/W$  and  $L'/W$ , with  $\beta$  and  $b$  assumed to be fixed. When comparisons are made of controls at different deflections, however, this will not necessarily be true and the effects of the  $\delta_1$  term in  $F_L$  and  $F_L'$  will have to be taken into account.

### Longitudinal Controls

In the analyses for longitudinal controls, controls located at the inboard, midspan, and tip positions on the wings were included. Figure 2 illustrates these positions together with the limiting Mach line locations for each position. As shown in figure 2 for tip controls considered in the present paper, the Mach lines from the control root chords did not cross either the wing root chords or the wing tips. For midspan controls, the Mach lines from the controls did not cross the wing root chords or the wing tips and the Mach lines from the control root chords did not cross the control tips. For inboard controls, the Mach lines from the control tips did not cross either the wing root or wing tip chords and the Mach lines from the control root chords did not cross the control tips. It will be noted that the present paper includes results for controls having root chords coincident with the wing root chords, whereas the data presented in reference 1 are limited to controls for which the innermost Mach lines do not cross the wing root chords. To obtain the characteristics of controls located adjacent to the wing root chords, reflection planes, which would be expected to approximate the effects of fuselages in practice, were assumed to be located at the wing root chords; loading parameters for the inboard conical flow regions of these controls were obtained from figure 7 of reference 1.

In the analyses, a range of control shapes and sizes capable of producing a fixed lift-coefficient slope of  $\frac{C_{L\delta}}{A} = .0.0001$  was determined for each control position. Values of the parameter  $F_L$  were calculated for these controls and are presented in figure 3 as functions of the various control plan-form parameters. The sketches at the right of the charts illustrate the hinge line and trailing-edge sweep angles corresponding to the various curves in the accompanying charts when  $\beta$  is equal to 1.0 ( $M = \sqrt{2}$ ) and are intended only as an aid in orienting the reader.

Although the value of the lift-coefficient slope used in the calculations for the charts of figure 3  $\left(\frac{C_{L\delta}}{A} = 0.0001\right)$  is quite arbitrary, these charts have a wide range of application because for a given control shape and wing the value of  $C_{L\delta}/A$  is directly proportional to the square of the control span and the value of  $L/H$  is inversely proportional to the control span. By use of such proportions, the following equations for extending the data of figure 3 to include other values of lift coefficient slope are simply derived:

$$\left(\frac{2b_f}{b}\right)_1 = \left(\frac{2b_f}{b}\right)_0 \sqrt{\frac{(C_{L\delta}/A)_1}{(C_{L\delta}/A)_0}} \quad (4)$$

$$\left(\frac{L}{H}\right)_1 = \left(\frac{L}{H}\right)_0 \sqrt{\frac{(C_{L\delta}/A)_0}{(C_{L\delta}/A)_1}} \quad (5)$$

The subscript 0 refers to conditions when  $\frac{C_{L\delta}}{A} = 0.0001$  and subscript 1 refers to similar conditions for other arbitrary values of  $C_{L\delta}/A$ .

From equations (3) and (5), the equation for the ratio of lift to deflection work at values of  $C_{L\delta}/A$  other than 0.0001 becomes:

$$\left(\frac{L}{W}\right)_1 = \left(\frac{L}{W}\right)_0 \frac{(\delta_1)_0}{(\delta_1)_1} \sqrt{\frac{(C_{L\delta}/A)_0}{(C_{L\delta}/A)_1}} \quad (6)$$

#### Lateral Controls

Limitations of analyses.- In order to obtain some indication of the limitations of the analyses for lateral controls, resulting from the neglect of hinge moments due to rolling motions, sample calculations have been made for the steady rolling condition in which the wing damping moment is equal in magnitude to the rolling moment induced by aileron deflection.

Figure 4 presents theoretical ratios of hinge moment due to rolling to hinge moment due to aileron deflection, calculated by use of equations

from references 3 and 4, for  $60^\circ$  delta wings with aileron controls comprising various amounts of the wing tips. These configurations were chosen because the theoretical unit damping forces on such ailerons are unusually high (especially when the control spans are relatively small and the wing leading edges are subsonic) and hinge moments of such ailerons due to damping will therefore approach maximum values. For the configurations to be applicable in the present analysis (for unbalanced trailing-edge flap-type controls), it was necessary to assume the ailerons to be hinged about their leading edges, which coincide with the wing leading edges. Although these particular configurations are not of practical interest, they will probably give a reasonable indication of the maximum hinge moments due to rolling which might be obtained.

The data of figure 4 indicate that hinge moments due to rolling are quite sizeable and at first increase rapidly with control size. The rate of increase diminishes as the control size is increased, however, and the data appear to indicate that for extremely large controls, the ratio of hinge moment due to rolling to hinge moment due to deflection will approach a value equal to or slightly greater than 0.5. For controls comprising 10 to 15 percent of the wing area, which might be considered to be near the upper limit of the practical range for this type of control, it is shown that hinge moments due to rolling will cancel out about  $1/3$  of the hinge moment due to deflection. Remembering that the data of figure 4 are for the steady-roll condition, this value of  $1/3$  is probably a great deal higher than that which could be counted on in practice. Because of aircraft inertia, the rate of roll at the time the control reaches maximum deflection will be considerably less than the steady-roll rate. On the basis of time histories presented in references 5 and 6 a rate of  $1/2$  the steady roll rate would seem to be more nearly of the right order, in which case the hinge moment due to rolling would balance out only about  $1/6$  of the hinge moment due to deflection. It would thus appear that the analysis would not be seriously limited because of neglecting hinge moments due to rolling. Although comparisons of ratios of rolling moment to hinge moment due only to deflection might in some cases result in erroneous conclusions regarding the more desirable control, this will only happen when the ratios  $L'/H$  for the controls being compared are near equal. It should be remembered that the present analysis considers only unbalanced trailing-edge flap-type controls and that all-movable or balanced flap-type controls would require an entirely different analyses.

Method of analysis.- In the analyses for lateral controls it was not possible to treat control size and control location in the general manner used for longitudinal controls; consequently, the analyses are considerably more detailed than were those for longitudinal controls.

It would seem probable that controls located at the wing tips would in all cases have higher ratios of rolling moment to hinge moment and

rolling moment to deflection work because of their greater distance from the roll axis. Lift and hinge moment vary with location, however, and it is therefore necessary to determine whether this is true. In order to do this, the effects of spanwise location on the values of  $F_l$  have been calculated for a systematic range of control plan forms, and results of these calculations are presented in figure 5 where  $F_l$  is plotted against  $2y_f/b$ . The range of plan forms considered is indicated by the sketches at the right of the charts of figure 5 where the hinge line and trailing edge sweep angles are shown for  $\beta = 1.0$  ( $M = \sqrt{2}$ ). The most inboard control locations for which results are presented in figure 5 are such that the innermost Mach lines from the controls pass through the points of intersection of the wing-root chords and the wing trailing edges. The most outboard locations are such that the tip chords of the controls and of the wings are coincident, as shown in figure 2 for longitudinal controls. An examination of figure 5 reveals that in most cases controls located at the wing tips have higher (more negative) values of  $F_l$  than do the same controls when located farther inboard. In the few cases for which this is not true (on wings having sweptback trailing edges, figs. 5(d) and 5(e)), the advantages of slightly inboard locations are not large and it was therefore concluded that it would be sufficient in the present analysis to consider only controls located at the wing tips. It must be cautioned, however, that tip controls on wings with sweptback trailing edges will in some cases have considerably less effectiveness than controls located farther inboard, particularly in the transonic speed range (ref. 7). It should also be pointed out that, for some wing configurations, aeroelastic and viscous effects, which have not been considered in this analysis, might outweigh the advantages of tip location for the controls.

Figure 6 presents the results of calculations made to determine the values of  $C_{l\delta}/A$  and  $F_l$  for a range of control plan forms located at the tips of wings having various ratios of  $\frac{\tan \Lambda_{TE}}{\beta}$ . By use of the data presented in figure 6, it was possible to prepare the charts of figure 7 which show the variation of the parameter  $F_l$  with control plan-form parameters for controls which produce various fixed amounts of rolling moment. Values of  $\frac{C_{l\delta}}{A} = 0.0002, 0.0004, \text{ and } 0.0006$  were chosen as representative. As in figures 3, 5, and 6 the sketches at the right of the charts illustrate the hinge line and trailing-edge sweep angles in the accompanying charts when  $\beta$  is equal to 1.0 ( $M = \sqrt{2}$ ). It should be pointed out that, although tip chord was used to define control plan form in figures 5 and 6 for reasons of convenience in the necessary computations, aspect ratio has been used in the analysis charts of figure 7 because of its greater significance.

## RESULTS AND DISCUSSION

The discussion of the analysis charts for longitudinal and lateral controls (figs. 3 and 7) will each be divided into two parts. The first part of the discussion for longitudinal controls will deal with controls having maximum ratios of lift to hinge moment and the second part will deal with controls having maximum ratios of lift to deflection work. The division for lateral controls will be similar.

## Ratios of Lift to Hinge Moment

General.- It will be noticed that little data are presented in figure 3 for low-aspect-ratio inboard controls and for low-aspect-ratio midspan and tip controls on wings having sweptback trailing edges. This lack of data results from the limiting Mach line locations which are shown in figure 2 and have been previously discussed.

Effects of spanwise location.- From a comparison of the curves in the charts for the inboard position with those for the midspan position, it can be seen that in no case does an inboard control have a greater value of  $F_L$  than does a midspan control having the same plan form. This result might be expected since inboard controls have been assumed to be located adjacent to reflection planes and any portion of the loading normally carried by the adjacent wing which is reflected back onto the control would increase the hinge moment and probably result in lower values of  $L/H$ . It should be pointed out, however, that for high-aspect-ratio untapered controls and for inversely tapered controls having small root chords, the adverse effects of the reflection planes are not large.

In the charts for the midspan and tip-control positions, it will be noticed that, if values of  $\beta_{A_F}$  less than 1.0 (which seem impractically small) are neglected, the maximum value of  $F_L$  shown on each curve occurs at the maximum value of  $\beta_{A_F}$ . From a comparison of the curves for the midspan and tip positions, it can be seen that values of  $F_L$  at the maximum values of  $\beta_{A_F}$ , on corresponding curves, are in all cases for the midspan position equal to or higher than those for the tip position.

One other general group of controls which should be discussed is full-span controls. The loading of a full-span control having any particular shape would be obtained by assuming a reflection plane to be located adjacent to the root chord of a tip control having the same plan form and making a corresponding correction to the loading of the tip control. Since comparisons of inboard and midspan control positions have indicated that reflection planes, if having any effect, decrease

the values of  $L/H$ , full-span controls would be expected to have values of  $L/H$  equal to or less than those for tip controls.

It thus appears that values of  $L/H$  for midspan controls will always be equal to or higher than those for similar controls at other locations. Consequently, the analysis for determining the plan forms of controls having maximum values of  $L/H$  will be limited to the consideration of controls located at midspan positions.

Untapered controls.- The charts for midspan controls show maximum values of  $F_L$  in most cases for untapered controls having values of  $\beta A_F$  equal to 8.0 (the upper limit of the calculations). The curves for untapered controls, if extended to higher aspect ratios, would be expected to show still higher values of  $F_L$  because the lift has been fixed, and higher aspect-ratio controls would necessarily have smaller chords and, consequently, smaller moment arms and hinge moments. It therefore appears that maximum values of  $L/H$  will be obtained by use of very high-aspect-ratio untapered controls. In practice, however, it will not in most cases be possible to obtain sufficient lift by use of extremely high-aspect-ratio controls. When the lift requirements are sufficiently high to require the use of moderate- and low-aspect-ratio controls, the data of figure 3 show that untapered controls will probably not have maximum values of  $L/H$ .

Tapered controls.- In the charts of figure 3, the maximum aspect ratios shown for tapered controls, represented by points farthest to the right, are the maximum aspect ratios possible for the particular combination of hinge line and trailing-edge sweep and consequently represent triangular control plan forms. The only exceptions are the curves for  $a = 0.80$  in figure 3(e) where the aspect ratio corresponding to triangular controls is beyond the range of the calculations.

It should be pointed out, as previously mentioned, that in comparing controls having various hinge-line sweep angles to determine which sweep angle gives the maximum values of  $L/H$ , comparisons must be made on the

basis of  $\frac{F_L}{\cos \Lambda_{HL}}$  rather than simply  $F_L$  as plotted in figure 3. When

$a$ , that is  $\frac{\tan \Lambda_{HL}}{\beta}$ , is equal to zero,  $\cos \Lambda_{HL}$  is equal to 1.0 and

$\frac{F_L}{\cos \Lambda_{HL}}$  is equal to  $F_L$ . With increases in the absolute value of  $a$ , however,  $\cos \Lambda_{HL}$  decreases and  $\frac{F_L}{\cos \Lambda_{HL}}$  increases. Consequently,

comparisons on the basis of  $L/H$  must be made by shifting the curves



for finite values of  $a$  down, the amount to depend on the value of  $a$  and Mach number since  $\cos \Lambda_{HL} = \frac{1}{\sqrt{1 + \beta^2 a^2}}$ .

The charts for the midspan control positions in figures 3(a) and 3(b) show that tapered controls having maximum values of  $L/H$ , for use on wings with sweptforward trailing edges, will have inversely tapered triangular plan forms with highly sweptforward hinge lines ( $a = -0.95$ ). The data for the sweptforward trailing-edge case (fig. 3(b)) can be used to illustrate the effect of the  $\cos \Lambda_{HL}$  term in the parameter  $F_L$ . It can be seen that the value of the parameter  $F_L$  is greater for the untapered control ( $a = -0.40$ ) having  $\beta A_F = 8.0$  than for the inversely tapered control having  $a = -0.95$  and  $\beta A_F = 3.6$ . It can be shown by use of equation 1(b), however, that at Mach numbers greater than 1.29 the effect of the  $\cos \Lambda_{HL}$  terms is such that the inversely tapered control has the higher value of  $L/H$ .

Figure 3(c) shows that, for wings having unswept trailing edges, the plan forms of tapered controls having maximum values of  $L/H$  are triangular in shape and have highly swept hinge lines. The figure shows identical values of  $L/H$  for normally and inversely tapered controls.

For wings having sweptback trailing edges, maximum values of  $L/H$  are shown for controls with triangular plan forms of normal taper. It will be interesting to note that ratios of  $L/H$  for the more desirable tapered and untapered controls located at the midspan position are not a great deal larger than ratios of  $L/H$  for controls having the same plan forms but located at the tip or inboard positions.

Without knowing the wing geometry, the maximum control span which may be used, the Mach number, and the required value of  $CL_{\delta}/A$  it is not possible to specify when tapered controls will have higher values of  $L/H$  than untapered controls. When these parameters are known, however, it will be quite simple, by use of the charts of figure 3, to determine whether untapered or tapered controls will provide greater values of  $L/H$ . With the insertion in equation (4) of the maximum control span which may be used  $(2b_f/b)_1$  and the required value of  $(CL_{\delta}/A)_1$  a value of  $2b_f/b$  corresponding to the lift-coefficient slope of figure 3  $\left(\frac{CL_{\delta}}{A} = 0.0001\right)$  will be obtained. The value of  $L/H$ , indicated by the appropriate chart of figure 3 for an untapered control having this value of  $2b_f/b$ , can then be compared with values of  $L/H$  for tapered controls having this span or smaller spans and smaller aspect ratios.

### Ratios of Lift to Deflection Work

Effects of spanwise location.- In determining control locations for which ratios of lift to deflection work are maximum, the procedure is the same as in the case where hinge moment is the criterion. This is true because comparisons are made between controls of constant shape and constant  $C_{L\delta}/A$ , in which cases maximum values of  $F_L$  correspond to maximum values of both  $L/H$  and  $L/W$ . The conclusions regarding control locations for maximum values of  $L/W$  would therefore be the same as those regarding control locations for maximum values of  $L/H$ ; that is, values of  $L/W$  for controls located in midspan positions will always be equal to or higher than values for similar controls at other locations.

Effects of control plan form.- In determining control plan forms for maximum ratios of lift to deflection work, the use of the charts of figure 3 is considerably more simple than was the case in the analysis dealing with hinge moment because comparisons are made on the basis of  $F_L$  rather than  $\frac{F_L}{\cos \Lambda_{HL}}$ .

All the charts for the midspan control position in figure 3 show maximum values of  $F_L$ , and therefore maximum ratios of lift to deflection work, for untapered controls with values of  $\beta A_F = 8.0$ . Since values of  $L/W$  would increase with increasing values of  $\beta A_F$ , as discussed in the section dealing with hinge moment, it is concluded that untapered controls of maximum aspect ratio will have maximum values of  $L/W$ . (It is of interest to note that this conclusion is similar to a result obtained in the analysis for the incompressible case of reference 2 wherein it is stated that flaps should be of almost constant chord and should be as long and narrow as compatible with structural and other design considerations.) It must be remembered, however, that values of  $\beta A_F$  above 8.0 would correspond to impractically high aspect ratios at relatively low supersonic Mach numbers.

It might be well to note that the advantages of untapered controls over tapered controls decrease as the wing trailing-edge sweep (either sweepforward or sweepback) is increased. Also, the effects of control location are relatively small for the high-aspect-ratio untapered controls.

Effects of control size.- The effect of control size on the value of  $L/W$  can be readily determined from equation (6). For a given amount of required lift and a given control shape, control lift-coefficient slopes are inversely proportional to control deflection, and equation (6) can be rewritten:

$$\left(\frac{L}{W}\right)_1 = \sqrt{\frac{C_{L\delta 1}}{C_{L\delta 0}}} \left(\frac{L}{W}\right)_0 \quad (6a)$$

It can be seen from equation 6(a) that the ratio of lift to deflection work for controls of similar shape is proportional to the square root of lift-coefficient slope; also to the square root of control area since  $C_{L\delta}$  is directly proportional to control area.

#### Ratios of Rolling Moment to Hinge Moment

Effects of control size.- From a comparison of the charts for the three values of  $C_{L\delta}/A$ , it will be seen that the ratios of rolling moment to hinge moment increase with decreasing values of  $C_{L\delta}/A$  and, consequently, with decreasing control size. This is a logical result because the ratio of rolling moment to hinge moment is essentially a ratio of moment arms, and the ratio of moment arms increases as the size of a control of given shape decreases. This result is significant because it indicates that control hinge moments can be appreciably reduced by using smaller controls and larger deflections.

Untapered controls.- The curves for untapered controls in figure 7 show that the rate of increase in  $F_l$  with control aspect ratio increases quite rapidly as the value of  $C_{L\delta}/A$  decreases with the consequence that high-aspect-ratio untapered controls compare favorably with the tapered controls at  $\frac{C_{L\delta}}{A} = 0.0002$ . This trend appears to indicate that untapered controls will have higher ratios of rolling moment to hinge moment than tapered controls when the rolling requirements are sufficiently low (values of  $C_{L\delta}/A$  somewhat less than 0.0002).

The aspect ratios at which maximum values of  $L'/H$  occur for untapered controls with  $\frac{C_{L\delta}}{A} = 0.0002$  are beyond the range of the calculations. For  $\frac{C_{L\delta}}{A} = 0.0004$  and 0.0006, however, untapered controls having maximum values of  $L'/H$  are shown to have spans roughly between 60 and 80 percent of the semispan of wings having unswept and sweptback trailing edges, regardless of the value of  $\beta A_f$ . Controls having spans between 60 and 80 percent of the wing semispan are also shown to have maximum values of  $L'/H$  on wings having sweptforward trailing edges if values of  $\beta A_f$  less than 4 are neglected. It thus appears that span is

the important parameter for defining the plan forms of untapered controls having maximum values of  $L'/H$ , except possibly for wings having swept-forward trailing edges.

The indication that span is the important parameter can be somewhat substantiated by means of plane geometry if it is assumed that the controls are uniformly loaded and that no loads are carried on the wing (making it possible to work with simple area moments). It can be shown that, for any arbitrary rolling moment, the ratio of rolling moment to hinge moment for a control located at the wing tip increases with control span until it reaches a maximum value when the control span is two-thirds of the wing semispan. It seems logical that this type of analysis would be applicable, except for low-aspect-ratio controls or low Mach numbers in which cases the conical-flow regions are quite large and cannot be neglected.

Tapered controls - sweptforward trailing edges. - Figures 7(a) and 7(b) show that on wings having sweptforward trailing edges, inversely tapered controls having triangular plan forms, and highly sweptforward hinge lines ( $a = -0.95$ ) will in practically all cases provide maximum values of  $L'/H$ .

Tapered controls - trailing edges unswept. - The data in the chart for  $\frac{C_{l\delta}}{A} = 0.0002$  in figure 7(c) show considerably greater values of  $F_l$  for high-aspect-ratio untapered controls than for the tapered controls ( $a = \pm 0.6$ ). At Mach numbers greater than 1.91, however, the normally and inversely tapered triangular controls both have higher ratios of rolling moment to hinge moment than does the untapered control of  $0.65b/2$  span (see eq. 1(b)). On the basis of figure 7(c), it would be expected that triangular plan forms having absolute values of  $a$  greater than 0.6 would have higher values of  $L'/H$  than untapered controls of  $0.65b/2$  span at Mach numbers considerably less than 1.91. It is therefore concluded that for  $\frac{C_{l\delta}}{A} = 0.0002$  and at moderate and high Mach numbers, maximum values of  $L'/H$  will be obtained by use of triangular plan forms and highly swept hinge lines. Although normally tapered triangular plan forms have somewhat higher values of  $L'/H$  than do inversely tapered triangular plan forms, it is probable that, because of structural considerations of the supporting wings, the inversely tapered controls will be more practical when the hinge lines are highly swept. The data in the charts for  $\frac{C_{l\delta}}{A} = 0.0004$  and  $0.0006$ , although showing very little difference in values of  $F_l$  for controls having hinge lines sweptforward and sweptback ( $a = 0.6$  and  $-0.6$ ), indicate

maximum values of  $L'/H$  for inversely tapered controls having triangular plan forms and highly sweptforward hinge lines.

Tapered controls, trailing edges swept back.- For wings having sweptback trailing edges, figures 7(d) and 7(e), in general, show maximum values of  $L'/H$  for normally tapered controls ( $a = 0.8$ ). Since the effects of normal taper on the area distribution of controls is such that reduced values of  $L'/H$  would be ordinarily expected, it seems probable that the advantage of normally tapered controls results from their larger regions of conical flow. The importance of such regions can be surmised from the charts for  $\frac{C_{l\delta}}{A} = 0.0004$  and  $0.0006$ , where optimum values of  $\beta A_F$  are near the minimum values shown on the curves. (The minimum values shown on these curves, as throughout figure 7, are near the values at which the Mach cones from the inboard-control parting lines intersect the control tips, corresponding to comparatively large regions of conical flow.) Going from minimum to maximum values of  $\beta A_F$ , the charts for  $\frac{C_{l\delta}}{A} = 0.0002$  show that values of  $L'/H$  for normally tapered controls first increase to maximum values and then decrease. Because the areas of the conical-flow regions decrease consistently with increasing control aspect ratio, there is evidently some parameter more important than the areas of the conical-flow regions which causes values of  $L'/H$  to increase as values of  $\beta A_F$  are increased. This parameter is probably control-area distribution because it has been shown for untapered controls that increased values of  $L'/H$  can be obtained by increasing the aspect ratios and spans of controls having spans of less than about two-thirds of the wing semispan. Figures 7(d) and 7(e), therefore, appear to indicate that plan forms of tapered controls on wings having swept-back trailing edges, for which maximum values of  $L'/H$  exist, are dependent on the interrelated parameters, control-area distribution and conical-flow area, and cannot be generally specified.

As a matter of practical interest, it should be mentioned that differences between the hinge-line ( $a = 0.8$ ) and trailing-edge sweep angles corresponding to the Mach number range between 1.3 and 2.5 are roughly between  $15^\circ$  and  $19^\circ$  in figures 7(d) and between  $7^\circ$  and  $8^\circ$  in figure 7(e). These differential angles are sizeable, and it is probable that on wings having relatively small differences between the leading and trailing-edge sweep angles, smaller differences will be of more practical interest. If somewhat smaller differences are considered, corresponding to values of  $a$

which are nearer to values of  $\frac{\tan \Lambda_{TE}}{\beta}$ , figures 7(d) and 7(e) indicate that advantages of tapered controls over untapered controls will, in general, be relatively small and at values of  $\frac{C_{l\delta}}{A} = 0.0002$  will probably be nonexistent.

### Ratios of Rolling Moment to Deflection Work

In using the charts of figure 7 to determine the plan forms of controls having maximum ratios of rolling moment to deflection work at a given value of  $C_{l\delta}/A$ , the various curves are compared directly (on the basis of  $F_l$ ). For controls having different values of  $C_{l\delta}/A$ , however, maximum values of  $F_l$  will not necessarily correspond to maximum values of  $L'/W$  (since different control deflections will be required to produce a fixed rolling moment) and comparisons of such controls must therefore be made on the basis of values of  $F_l/\delta_1$  rather than  $F_l$ .

Effects of control plan form.— With the exception of figure 7(a), all the data for  $\frac{C_{l\delta}}{A} = 0.0002$  in figure 7 show maximum values of  $L'/W$  for high-aspect-ratio untapered controls. In figure 7(a), higher values of  $L'/W$  are shown for inversely tapered triangular controls with highly swept hinge lines. The data for the three values of  $C_{l\delta}/A$  in figure 7(a) appear to indicate, however, that high-aspect-ratio untapered control plan forms will have maximum values of  $L'/W$  at values of  $C_{l\delta}/A$  somewhat less than 0.0002. It might therefore be concluded that for sufficiently small controls, maximum values of  $L'/W$  will in all cases be obtained by use of high-aspect-ratio untapered plan forms. The spans of untapered controls for maximum values of  $L'/W$ , as discussed for maximum values of  $L'/H$ , would probably be of the order of two-thirds of the wing semispan.

For controls having values of  $\frac{C_{l\delta}}{A} = 0.0004$  and 0.0006, the data of figures 7(a) and 7(b) for wings with sweptforward trailing edges show maximum values of  $L'/W$  for inversely tapered controls having triangular plan forms. For wings having unswept trailing edges, the effects of plan form on values of  $L'/W$  are shown in figure 7(c) to be relatively small. Untapered controls with spans of about two-thirds of the wing semispans, however, are shown to have values of  $L'/W$  which are equal to or greater than those for other control plan forms. Figures 7(d) and 7(e) show maximum values of  $L'/W$  on wings having sweptback trailing edges for normally tapered controls with values of  $\beta A_T$  between 3 and 5. As mentioned in the analysis dealing with hinge moments, the differences between hinge-line and trailing-edge sweep angles for the normally tapered controls of figures 7(d) and 7(e) will for many applications be impractically high; and for controls having values of  $\beta$  near values of  $\frac{\tan \Lambda_{TE}}{\beta}$ , which are probably of more practical interest, the data indicate that values of  $L'/W$  would be little if any higher than those for untapered controls with spans of about two-thirds of the wing semispan.

For purposes of comparison, it is of interest at this point to note the results obtained in the analysis of plan form for the incompressible case (ref. 2). These results are: The shape of ailerons for minimum deflection work is of maximum width near the wing tip and has a slight convex curvature as it tapers to zero chord at the center of the wing (somewhat similar to the sweptforward trailing-edge case of the present analysis). Partial span ailerons should be sections of these shapes and should include the regions of maximum chord. The ailerons should be as long and narrow as compatible with structural and other design considerations.

Effects of control size.— In using the data for the different values of  $C_{l\delta}/A$  in figure 7 to determine effects of control size on the value of  $L'/W$ , it was necessary to use a fixed value of  $C_l/A$  as a basis for comparison because of the  $\delta$  term in  $F_l$ . A value of  $\frac{C_l}{A} = 0.0006$  was arbitrarily chosen for which values of  $\delta_1 = 3, 1\frac{1}{2}$ , and 1 are required, respectively, for controls with values of  $\frac{C_{l\delta}}{A} = 0.0002, 0.0004$ , and 0.0006. The comparisons were then made by dividing values of  $F_l$  from the various charts by corresponding values of  $\delta$ . Results of the comparisons for control plan forms previously discussed as having higher values of  $L'/W$  are presented in the following table:

Figure	$\frac{\tan \Lambda_{TE}}{\beta}$	a	$\beta A_F$	$\frac{2b_f}{b}$	$\frac{\beta L'}{2 W}$ at $\frac{C_{l\delta}}{A}$ :		
					0.0002	0.0004	0.0006
7(a)	-0.60	-0.95	5.7	varied	11.3	13.2	13.6
7(b)	-.40	-.95	3.6	varied	8.8	10.7	11.3
7(c)	0	0	varied	0.65	8.0	8.0	8.2
7(d)	.40	.40	varied	.60	9.1	8.8	8.9
7(e)	.60	.60	varied	.65	----	10.9	10.6

The data in the above table show that for wings having sweptforward trailing edges, there are appreciable increases in values  $L'/W$  with increasing size of inversely tapered controls. For wings having unswept and sweptback trailing edges, little effect of the size of untapered controls on values of  $L'/W$  is shown. It would thus appear, especially when the relieving effect of hinge moments due to rolling are considered, that larger controls would in most cases have somewhat higher ratios of rolling moment to deflection work than would smaller controls.

VARIATIONS WITH MACH NUMBER OF THE CHARACTERISTICS OF  
EXAMPLE LATERAL CONTROL CONFIGURATIONS

Conditions

Specifications.- It is very difficult from the analysis charts of figure 7 to visualize the characteristics of lateral control surfaces on aircraft operating over a speed range. In order to illustrate the variation with Mach number of the characteristics of some of the control plan forms which have been shown to be desirable, some example calculations have been made. The specifications used for the calculations are as follows: Wings having spans of 38 feet are to be equipped with aileron controls capable of producing rolling rates of 3.0 radians per second while operating at Mach numbers up to 2.25 at altitudes of 40,000 feet. Wings are to have trailing-edge sweep angles of  $-20^\circ$ ,  $0^\circ$ , and  $35^\circ$  with other plan-form variables unspecified. Combined deflections of ailerons on opposite wing panels are not to exceed  $30^\circ$ .

Determining values of  $C_{l8}/A$  required.- In figure 8 are presented, as a function of Mach number, wing-tip helix angles  $pb/2V$  corresponding to the above specified conditions of wing span, rate of roll, and altitude. In order to determine the rolling moments required to produce these wing-tip helix angles, it is necessary to know the wing damping-moment coefficients. Figure 9 presents the theoretical damping-moment coefficients for a broad range of wing plan forms and Mach numbers obtained by use of charts presented in references 4 and 8. In order to calculate the required rolling moments without fixing wing plan forms, it was necessary to make the simplifying assumption that damping-moment coefficients do not change with Mach number; and figure 10 has been prepared for the purpose of examining such an assumption. Figure 10 shows that damping coefficients of highly swept wings are relatively independent of Mach number and that damping coefficients of high-aspect-ratio wings are influenced to a greater extent by Mach number than the damping coefficients of low-aspect-ratio wings. From figure 10 it can be seen that results obtained in the present paper by assuming fixed damping-moment coefficients will be directly applicable to moderate and low-aspect-ratio wings having highly swept leading edges.

Rolling-moment coefficients corresponding to the wing-tip helix angle of figure 8 and to fixed damping-moment coefficients which were considered to be representative, on the basis of figure 9, are presented in figure 11. To determine rolling-moment-coefficient slopes corresponding to the values of  $C_{l8}/A$  presented in figure 11, it is only necessary to divide the values of  $C_{l8}/A$  by  $30^\circ$ . To provide a more practical example, however, some consideration should be given to the



effects of wing thickness, nonlinearities of control effectiveness with control deflection, and wing flexibility, which are known to result in actual values of control effectiveness which are considerably less than theoretical. It was shown by the illustrative example of reference 1 that, when an approximate thickness correction was applied, the effectiveness of a control on a 5-percent-thick wing was reduced to about 80 percent of that predicted by theory. On the basis of this example, and making an arbitrary allowance for nonlinearities, it was assumed that the effectiveness of controls on rigid wings would be 60 percent of that predicted by theory. It was further assumed, quite arbitrarily, that the effectiveness of controls on flexible wings would be 60 percent of that for controls on rigid wings and, consequently, 36 percent of theoretical. Estimated values of  $C_{l\delta}/A$  necessary to produce the required values of  $C_l/A$  (fig. 11) were obtained by use of the preceding assumptions and are presented in figure 12.

Control plan forms.- Rolling-moment-coefficient slopes and ratios of rolling moment to hinge moment were calculated through a Mach number range for several control plan forms on wings having trailing-edge sweep angles of  $-20^\circ$ ,  $0^\circ$ , and  $35^\circ$  and having damping-moment coefficients  $-C_{lp}/A$  of 0.08 (chosen as a mean value from fig. 9). For each configuration, calculations were made for untapered control plan forms having spans of approximately 50, 65, and 80 percent of the wing semispan and for tapered control plan forms having various hinge-line sweep angles. Calculations of  $L'/H$  were also made for representative control plan forms on wings having unswept trailing edges and having damping coefficients  $-C_{lp}/A$  of 0.03, 0.08, and 0.14. Results of the latter calculations were also used to illustrate the effects of control plan form and size on ratios of rolling moment to deflection work.

#### Results of $L'/H$ Computations

As can be seen in the charts for  $C_{l\delta}/A$  of figure 13, all of the control plan forms, for which results are presented, provide rolling moments equal to or greater than those which were estimated to be necessary to meet the required specifications (fig. 12).

Untapered controls.- Maximum values of  $L'/H$  for untapered controls are shown in figure 13 for controls having spans of about 65 percent of the wing semispan except at low Mach numbers on wings having trailing edge sweep angles of  $-20^\circ$ . In this case, the control having a span of 50 percent of the wing semispan provides slightly higher values of  $L'/H$ . These results illustrate the previously discussed conclusion that untapered control plan forms for maximum values of  $L'/H$  have spans of about  $2/3$  of the wing semispan except in cases of low control aspect ratios or low Mach numbers. It might be well to point out that the

advantage of the  $\frac{2(b)}{3(2)}$ -span flaps over the  $1/2$  and  $\frac{4(b)}{5(2)}$ -span flaps is small. If compared with flaps having spans of less than  $\frac{1(b)}{2(2)}$  or greater than  $\frac{4(b)}{5(2)}$ , however, the advantage of the  $\frac{2(b)}{3(2)}$ -span flaps would be expected to be greater.

It will be noticed in figure 13 that the values of  $L'/H$  for untapered controls increase as the controls are swept either forward or back. This result is probably due to the fact that the center of loading of the control, which remains near the same chordwise location regardless of sweep, is nearer the hinge line when the control is swept than when it is unswept. The above speculation can be somewhat substantiated because it can be shown that, by dividing the values of  $L'/H$  shown for the unswept case by appropriate  $\cos \Lambda_{TE}$  terms, values of  $L'/H$  for the swept controls can be roughly approximated. Thus the values of  $L'/H$  for the sweptback trailing edge are greater than those for the swept-forward trailing edge principally because of the greater sweep rather than the direction of sweep.

Tapered controls.- For wings having trailing-edge sweep angles of  $-20^\circ$  and  $0^\circ$  (fig. 13), consistent increases in the values of  $L'/H$  with  $\Lambda_{HL}$  are shown at any given Mach number. At the maximum design Mach number of 2.25, inversely tapered controls having hinge lines swept-forward  $60^\circ$  provide values of  $L'/H$  roughly 50 percent greater than those shown for untapered controls with  $\frac{2b_f}{b} = 0.65$ . At lower Mach numbers, a still greater advantage is shown for the inversely tapered controls. Simply stated, this means that the inversely tapered controls require at most only about two-thirds of the hinge moment required by untapered controls to produce the required rolling moments.

For wings having sweptback trailing edges, the tapered-control plan forms which are shown in figure 13 were chosen by fixing the hinge-line sweep angles at  $40^\circ$  and  $45^\circ$  (believed to be practical values for  $\Lambda_{TE} = 35^\circ$ ) and using figure 7 to estimate the more desirable aspect ratios. Results presented in figure 13 show that ratios of  $L'/H$  for the tapered controls are somewhat greater than for untapered controls at the lower Mach numbers but slightly less at the higher Mach numbers. Figure 13 indicates that greater values of  $L'/H$  could probably be obtained to substantially high Mach numbers with tapered controls if a considerably greater amount of hinge-line sweep were used. Aside from being structurally impractical, it would appear from figure 13 that such controls would have an extremely high rate of decrease in the values of  $L'/H$  with Mach number.

Effect of varying damping coefficients.— Because figure 7 indicates that control plan forms for maximum values of  $L'/H$  vary somewhat with the amount of rolling moment required, rolling moment and ratio of rolling moment to hinge moment have been calculated for example controls on wings having unswept trailing edges and having damping-moment coefficients  $-C_{lp}/A$  of 0.03, 0.08, and 0.14. For each damping-moment coefficient, calculations were made for untapered controls having spans of approximately 65 percent of the wing semispan and for inversely tapered triangular controls having hinge lines swept forward  $60^\circ$ . Results of these calculations are presented in figure 14.

It will be noted in figure 14 that data are presented for inversely tapered triangular controls at Mach numbers for which the hinge lines are swept behind the Mach lines (indicated by dashed lines). These data were obtained by use of equations presented in reference 3 and are of particular interest because they show that this type of control, which\* has been shown to have high values of  $L'/H$  at the higher Mach numbers, produces satisfactory rolling moments and increasing values of  $L'/H$  as the Mach number is decreased.

It can be seen in figure 14 that the ratios of rolling moment to hinge moment increase very rapidly as control size (with constant span for untapered controls) is decreased. This illustrates, as did figure 7, the advantage of using small controls and maximum practical deflections.

Figure 14 also shows that the advantage of the inversely tapered control over the untapered control decreases steadily with decreasing rolling-moment requirements until, for rolling moments corresponding to  $\frac{-C_{lp}}{A} = 0.03$ , the untapered control has a higher value of  $L'/H$  at a Mach number of 2.25. It might be pointed out, however, that the untapered control in this case has the very high, and perhaps somewhat impractical, aspect ratio of 16.5.

#### Results of $L'/W$ Computations

In order to illustrate some effects of control plan form and size on the ratios of rolling moment to deflection work, sample calculations were made for the controls shown in figure 14. It was assumed for the calculations that the rolling requirements of the controls were the same as in the previous examples and that the wing damping-moment coefficient  $-C_{lp}/A$  was 0.03. The upper chart of figure 15 presents the theoretical rolling-moment requirements for an assumed practical control effectiveness 36 percent of theoretical. (It should be mentioned that

the deflections necessary to produce the required rolling moments vary considerably with Mach number, as well as with control size, because of varying values of  $C_{l\delta}/A$ .) The lower chart of figure 15 presents the ratios of rolling moment to deflection work required to produce this rolling moment.

The data of figure 15 for the untapered controls illustrate the previously discussed conclusion regarding the effect of control size; that is, for controls having spans of about two-thirds of the wing semispan, ratios of rolling moment to deflection work are not appreciably influenced by control size. In consideration of hinge moments due to rolling, however, it is probable that maximum values of  $L'/W$  will in practice be obtained by use of the larger controls.

The data of figure 15 for the inversely tapered controls show sizeable increases in values of  $L'/W$  with control size. Similar results for this type of control on wings having sweptforward trailing edges, as previously mentioned, were indicated by the analysis charts.

#### SUMMARY OF RESULTS

Theoretical analyses have been made to determine the plan forms of unbalanced trailing-edge flap-type controls having minimum hinge moments due to deflection and requiring minimum work to overcome the hinge moments due to deflection at supersonic speeds. Ratios of lift and rolling moment to hinge moment and ratios of lift and rolling moment to deflection work at fixed values of lift and rolling effectiveness were used as bases for the analyses. Hinge moments due to angle of attack, damping in pitch and rolling, which are dependent on wing plan form and to varying degrees on complete aircraft configuration, have not been included in the present analyses and will have to be taken into account in applying results of these analyses to any particular wing.

Results of the analyses are summarized in table I and are as follow:

For longitudinal controls, maximum ratios of lift to hinge moment ( $L/H$ ) are obtained with untapered controls of maximum aspect ratio. In practice it will in many cases not be possible to obtain sufficient lift with high-aspect-ratio controls; and when moderate and low-aspect-ratio controls must be used, controls with triangular plan forms and highly swept hinge lines will have higher values of  $L/H$  than untapered controls. The plan forms of triangular controls having maximum values of  $L/H$  are inversely tapered for wings with sweptforward trailing edges and are normally tapered for wings with sweptback trailing edges. On wings with unswept trailing edges, direction of hinge-line sweep is of

little importance. For control plan forms having maximum values of  $L/H$ , control location is of little importance.

Maximum ratios of lift to deflection work are shown for untapered controls of high aspect ratio. In contrast with the results regarding hinge moment, untapered controls require less deflection work than tapered controls when the lift requirements are such that controls of moderate, and in some cases low, aspect ratio must be used.

For any given control shape the analysis for lateral controls illustrates the importance of using small controls with high deflections in obtaining maximum ratios of rolling moment to hinge moment ( $L'/H$ ). Control plan form, although secondary to control size, is also shown to be quite important. For wings having sweptforward or unswept trailing edges, inversely tapered controls having triangular plan forms and highly sweptforward hinge lines are shown to have maximum values of  $L'/H$ . For wings having sweptback trailing edges, control plan forms for maximum values of  $L'/H$  are dependent on the particular requirements and cannot be generally specified. Results indicate, however, that for such wings, little can be gained in practice by tapering the control. The spans of untapered controls having maximum values of  $L'/H$  are shown in most cases to be of the order of two-thirds of the wing semispan.

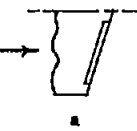
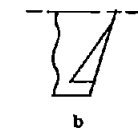
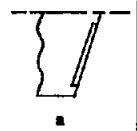
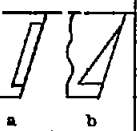
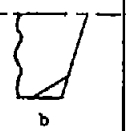
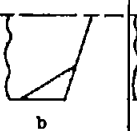
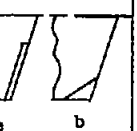
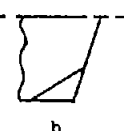
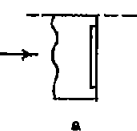
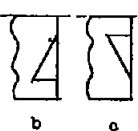
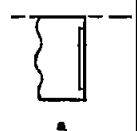
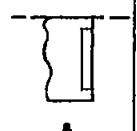
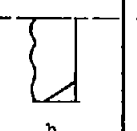
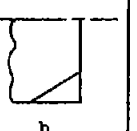
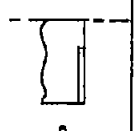
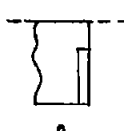
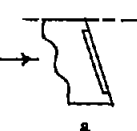
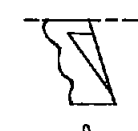
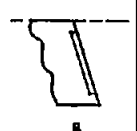
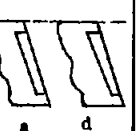
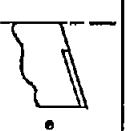
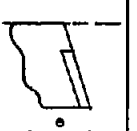
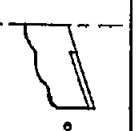
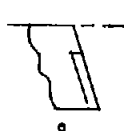
When the rolling requirements are low enough to permit the use of very small controls, maximum ratios of rolling moment to deflection work ( $L'/W$ ) are in all cases indicated for untapered controls of high aspect ratio. When more conventional control sizes are necessary, maximum values of  $L'/W$  on wings with sweptforward trailing edges are shown for inversely tapered controls with triangular plan forms. On wings with unswept and sweptback trailing edges, effects of hinge-line sweep are not of especial importance; and considering the more practical configurations, untapered controls with spans of about two-thirds of the wing semispan are indicated to have near maximum values of  $L'/W$ . Effects of control size on values of  $L'/W$  for these untapered controls are shown to be negligible. For the inversely tapered controls on wings with sweptforward trailing edges, however, values of  $L'/W$  are shown to increase appreciably with control size. Since hinge moments due to rolling increase with control size, it would thus appear that large controls would in most cases require less deflection work than smaller controls.

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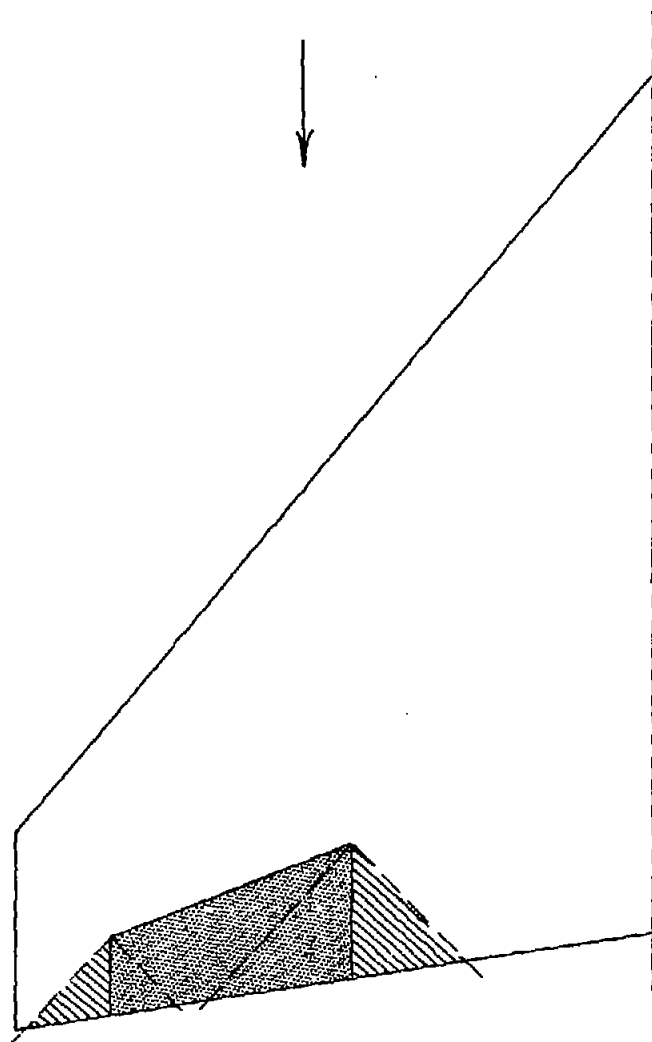
TABLE I.- PLAN FORMS OF CONTROLS GENERALLY SHOWN TO HAVE MAXIMUM RATIOS OF LIFT AND ROLLING MOMENT TO HINGE MOMENT  
DUE TO DEFLECTION AND MAXIMUM RATIOS OF LIFT AND ROLLING MOMENT TO WORK REQUIRED TO OVERCOME THE HINGE  
MOMENTS DUE TO DEFLECTION. (FOR MORE COMPLETE DESCRIPTIONS SEE SUMMARY OF RESULTS.)

Wing trailing-edge sweep condition	Plan forms for maximum L/H		Plan forms for maximum L/W		Plan forms for maximum L'/H		Plan forms for maximum L'/W	
	Small control	Large control	Small control	Large control	Small control	Large control	Small control	Large control
Sweptforward	 a	 b	 a	 a b (note 2)	 b	 b	 e b (note 5)	 b
Unswept	 a	 b	 a	 a	 b	 b	 e	 a
Sweptback (note 1)	 a	 c	 a	 a d (note 3)	 e (note 4)	 e (note 4)	 e	 e (note 6)

- a. High-aspect-ratio untapered control plan forms.  
b. Inversely tapered triangular control plan form, highly sweptforward hinge line.  
c. Normally tapered triangular control plan form, highly sweptback hinge line.  
d. Normally tapered control plan form.  
e. High-aspect-ratio untapered control plan form,  $\frac{2b_f}{b} \approx \frac{2}{3}$ .

Note: 1. Tip controls shown in some cases have less rolling effectiveness than similar controls located further inboard.  
2. L/W maximum for plan form b when wing trailing edge is highly sweptforward and low-aspect-ratio control is required.  
3. L/W maximum for plan form d when wing trailing edge is highly sweptback and moderate or low-aspect-ratio control is required.  
4. Plan form d will for some conditions of Mach number and trailing-edge sweep have higher values of L'/H.  
5. L/W maximum for plan form b when wing trailing edge is highly sweptforward.  
6. Plan form d will for some conditions of Mach number and trailing-edge sweep have higher values of L'/W.





— — — Mach lines



Surface of control



Portions of wing on which loads are induced by control deflection.



Figure 1.- Illustration of regions in which loading is influenced by the deflection of trailing-edge controls at supersonic speeds.



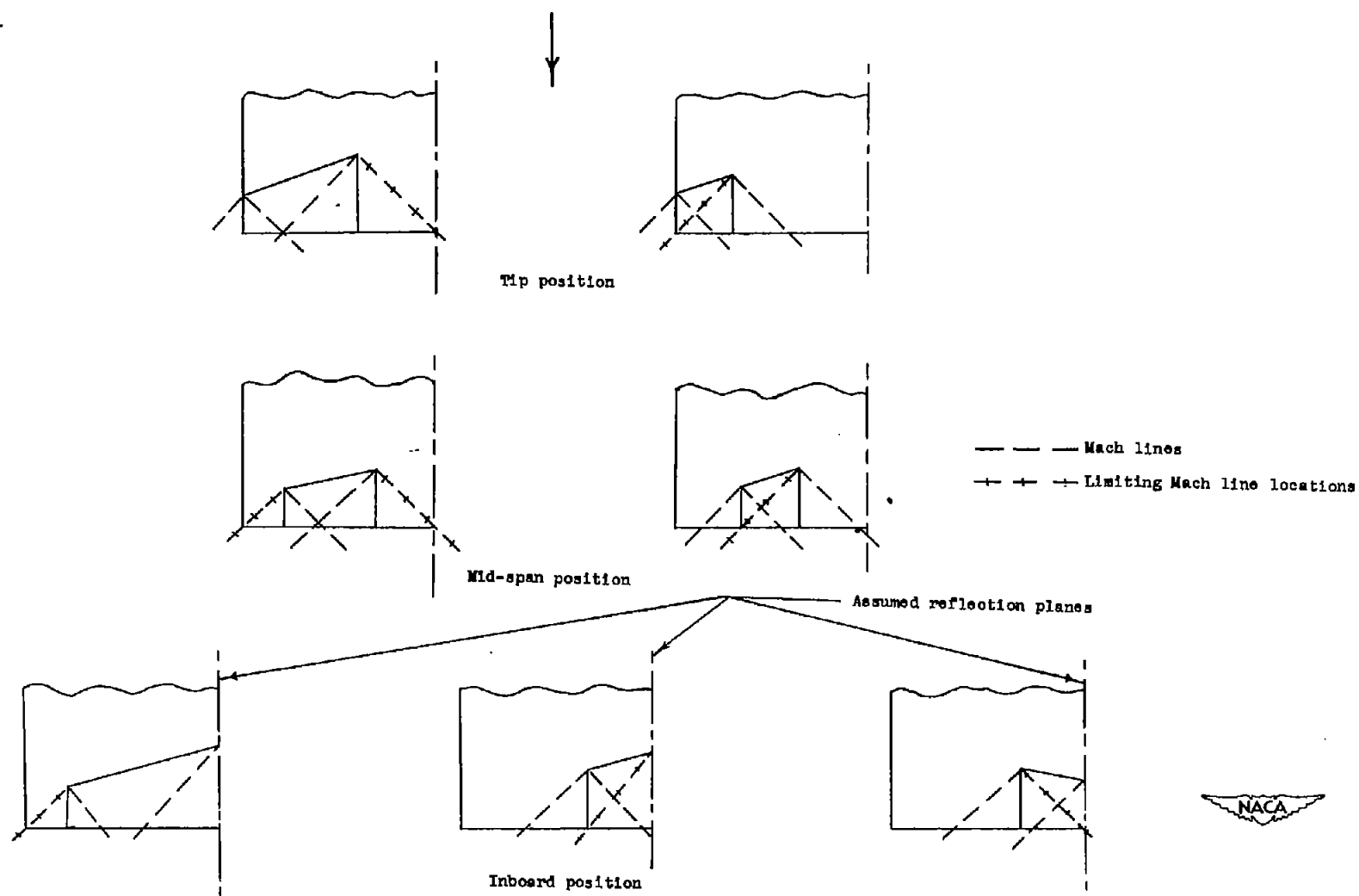


Figure 2.- Illustration of limiting Mach line locations for various positions of longitudinal control surfaces.

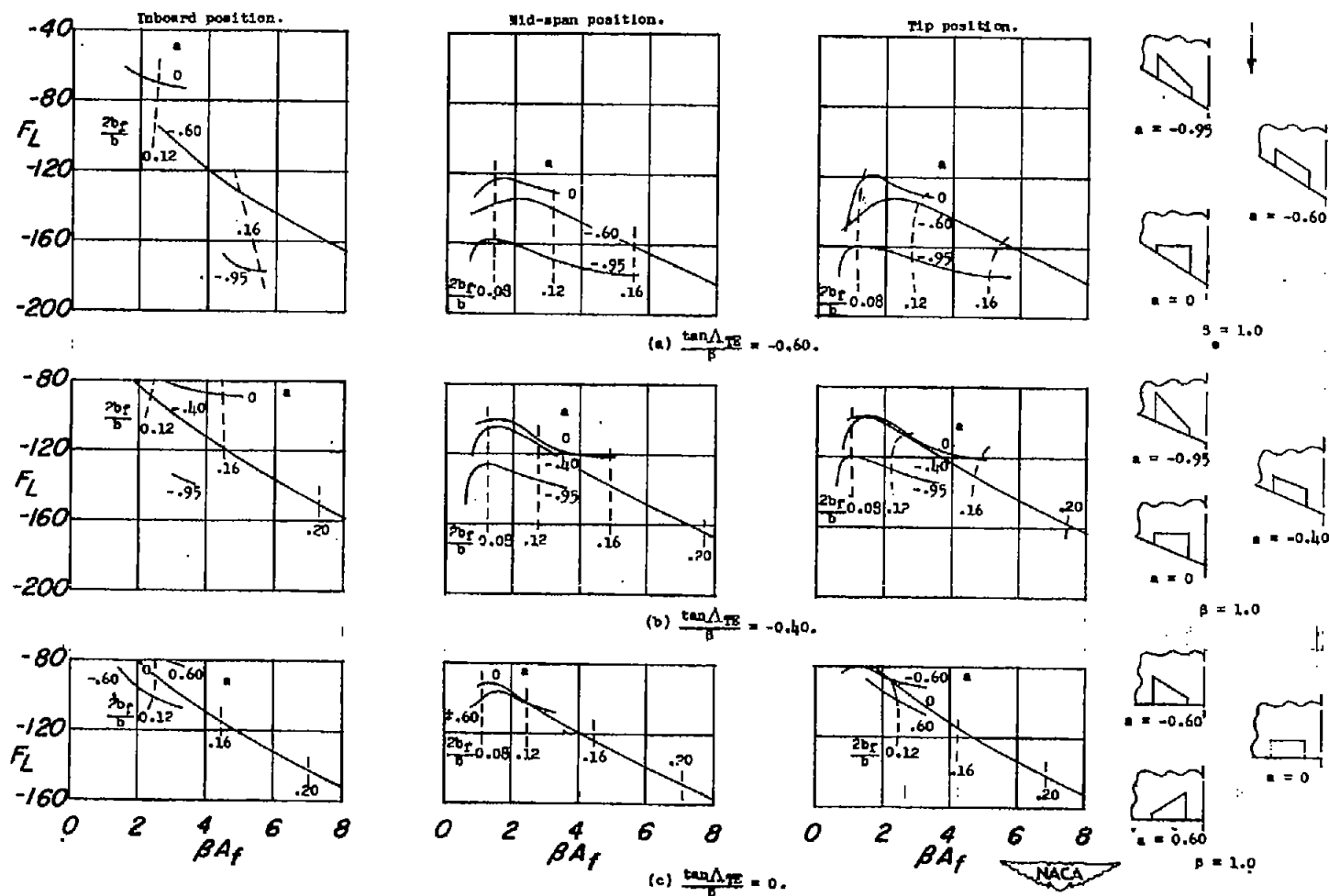


Figure 3.- Variations with control plan form and location of the

parameter  $F_L$  when  $\frac{C_{L\delta}}{A} = 0.0001$ .  $\left( F_L = \beta b \cos \Lambda_{HL} \frac{L}{H} = \frac{\delta_1}{57.3} \frac{\beta b}{2} \frac{L}{W} \right)$

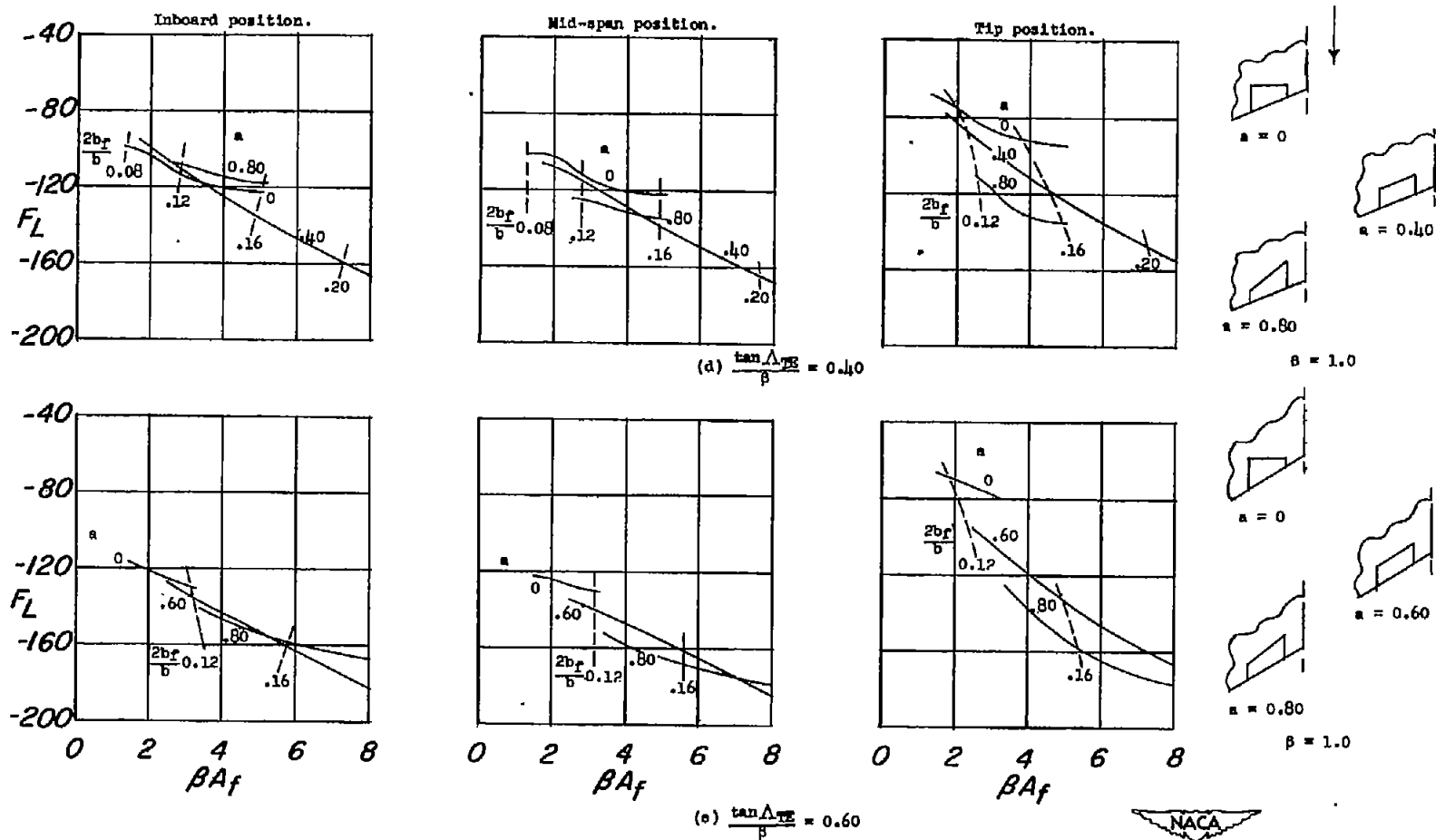


Figure 3.- Concluded.

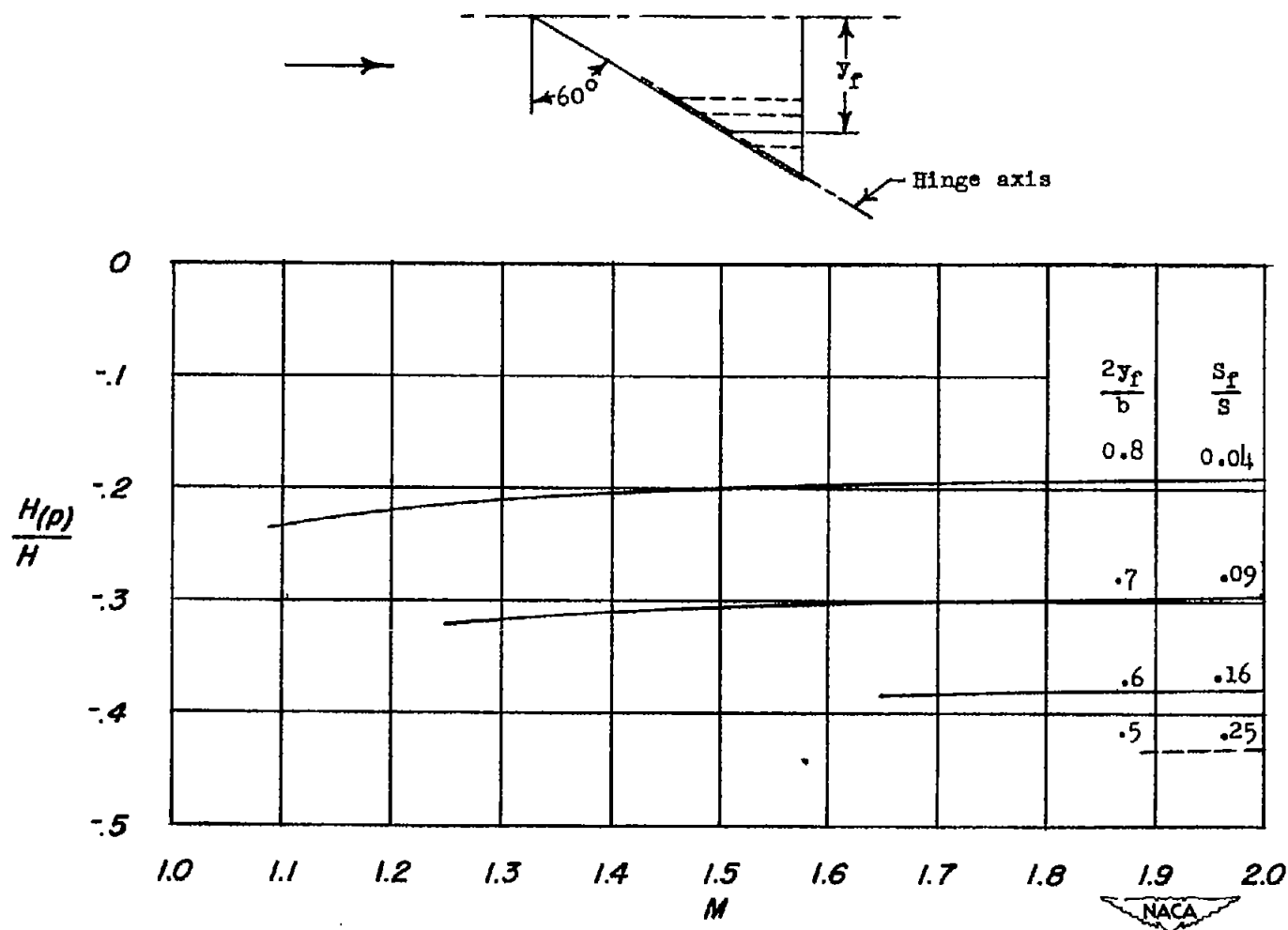


Figure 4.- Ratio of hinge moment due to steady rolling to hinge moment due to deflection for tip controls of various sizes on a  $60^\circ$  delta wing (rolling moment produced by control assumed equal to wing damping moment).

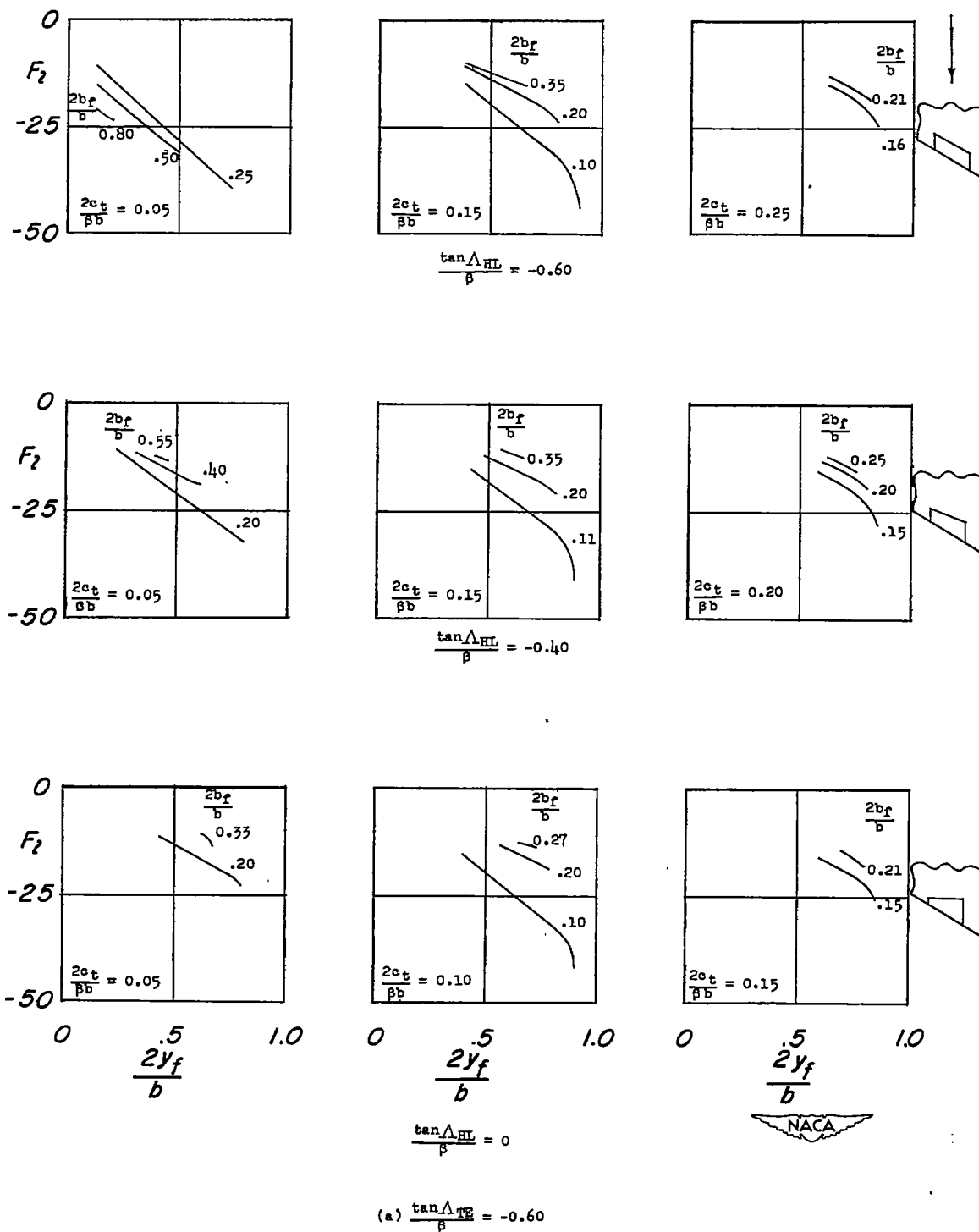


Figure 5.- Variations with spanwise location of the parameter  $F_L$  for various control plan forms.  $\left( F_L = \beta \cos \Lambda_{HL} \frac{L'}{H} = \frac{\delta_1}{57.3} \frac{\beta}{2} \frac{L'}{W} \right)$

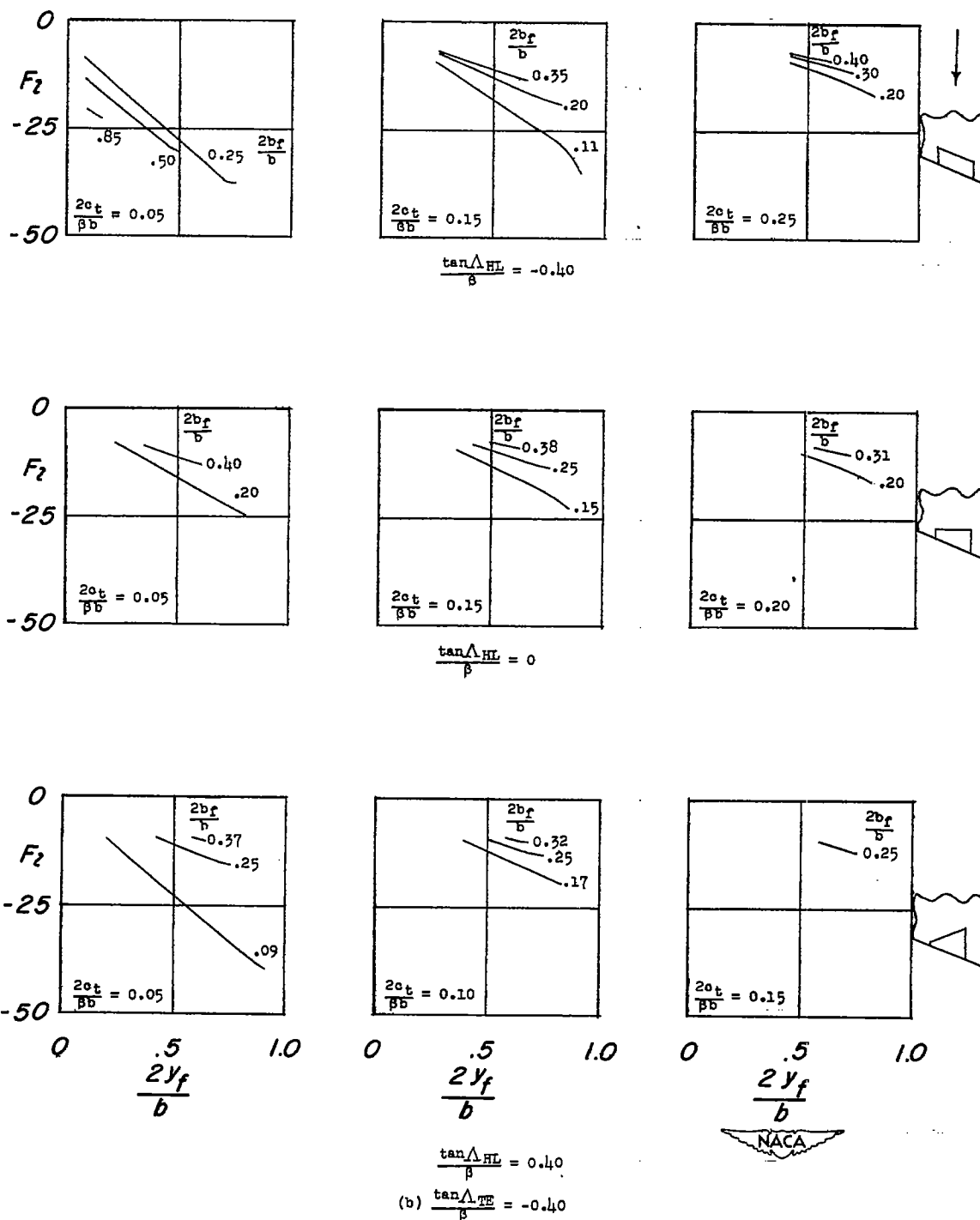
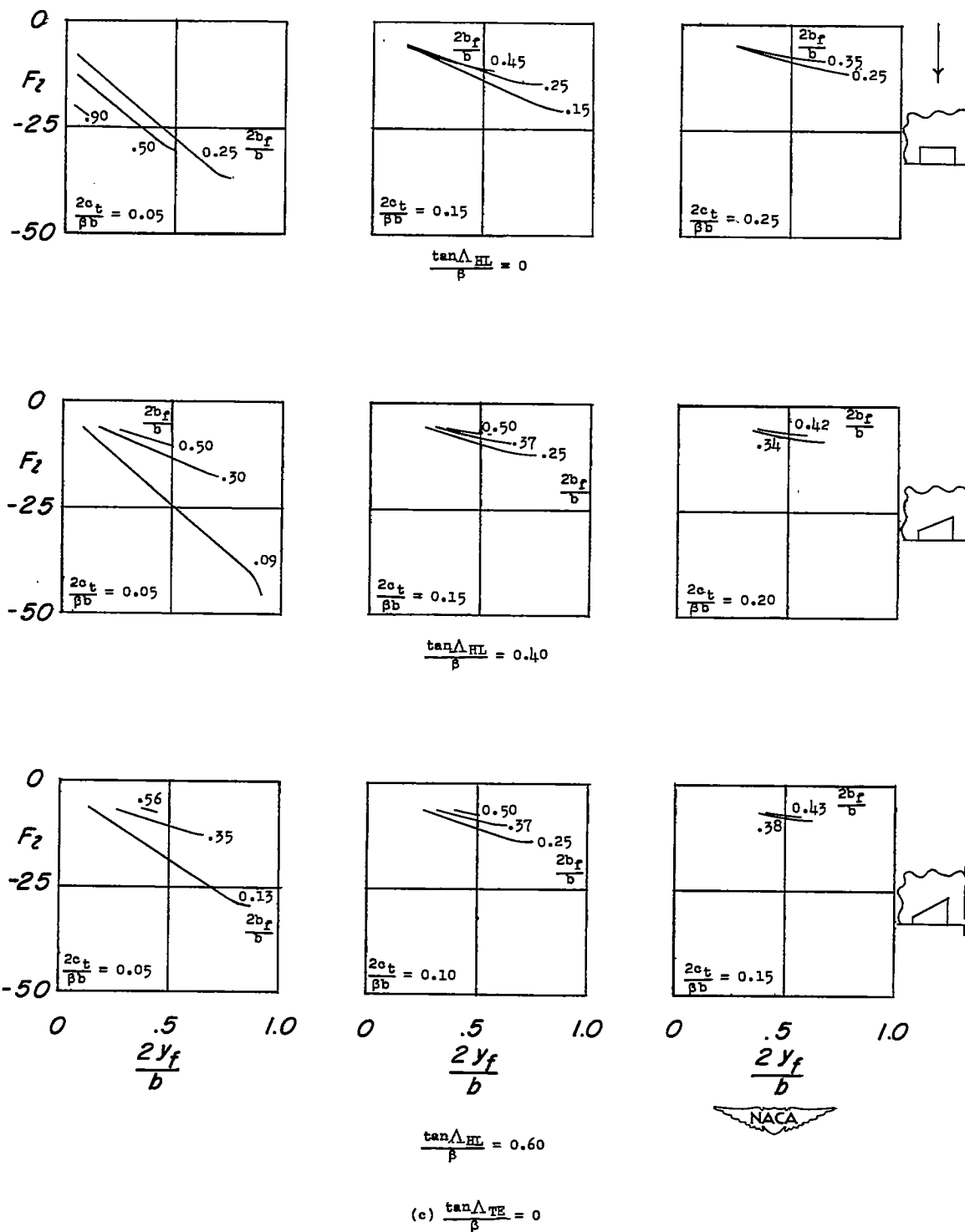


Figure 5.- Continued.

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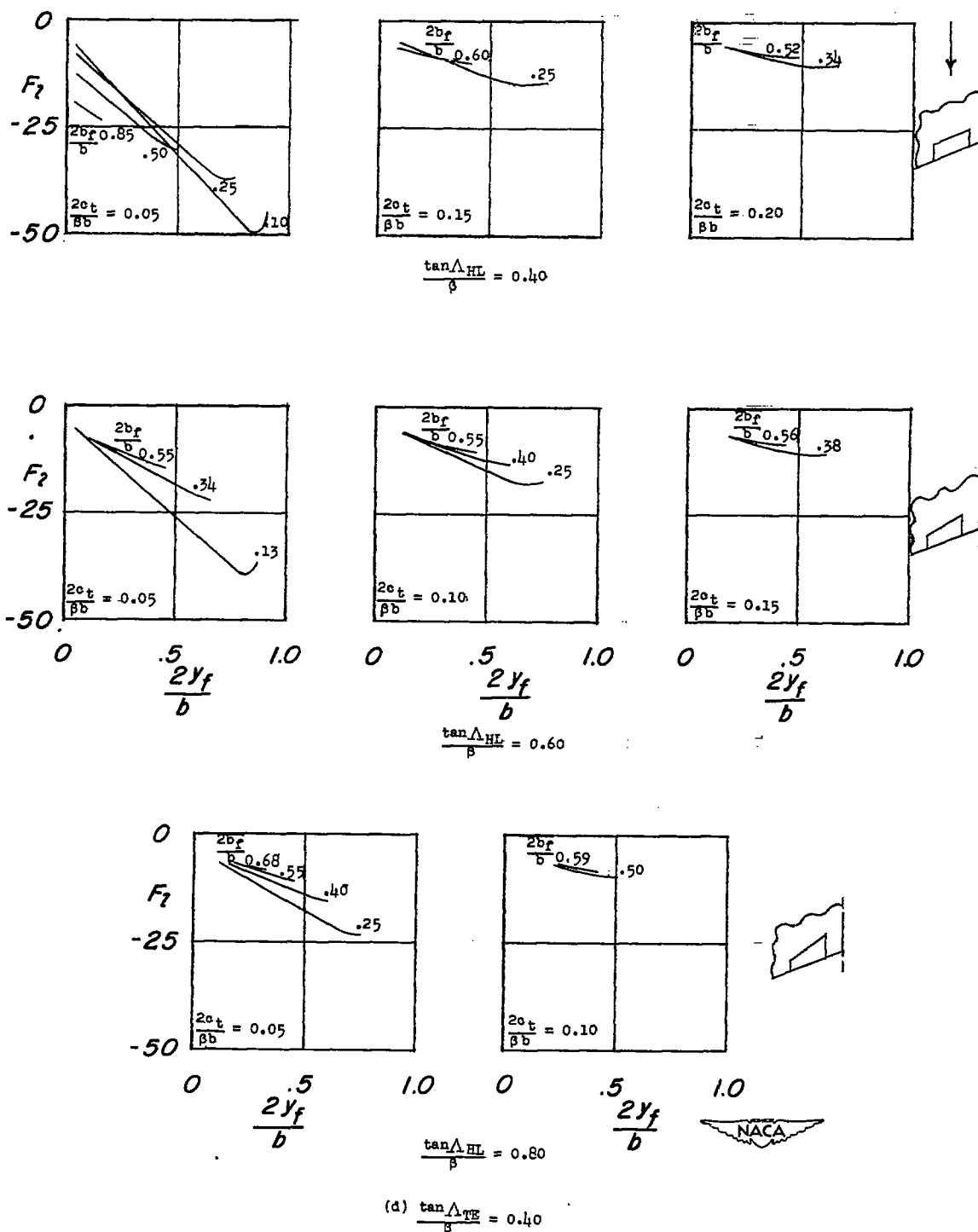
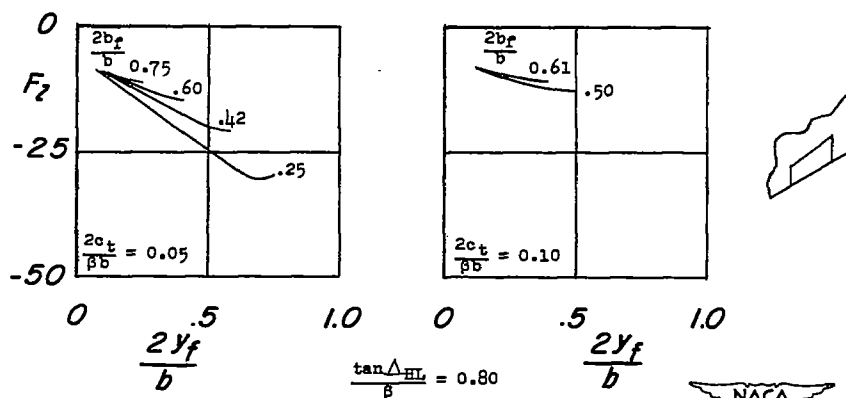
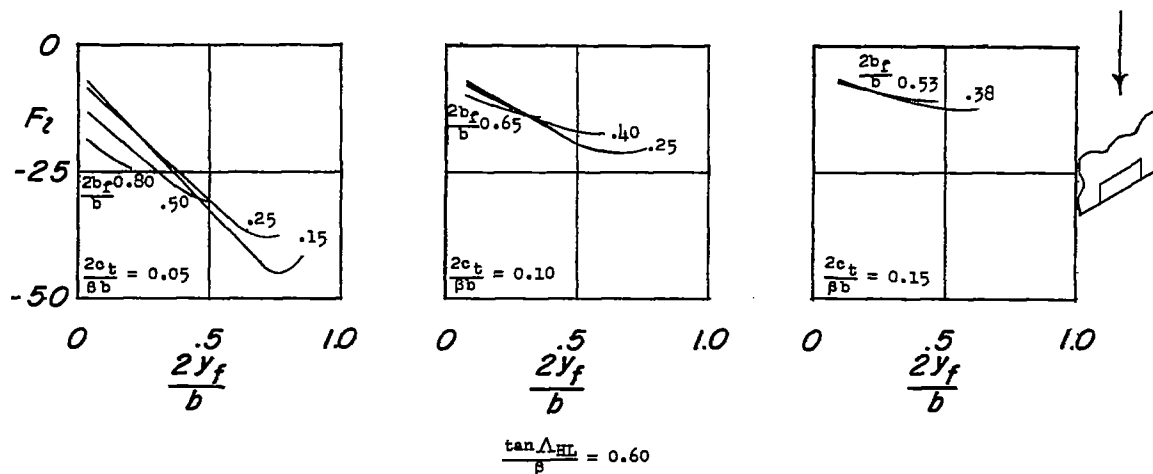


Figure 5.- Continued.





(e)  $\frac{\tan \Delta_{TR}}{\beta} = 0.60$

Figure 5.- Concluded.

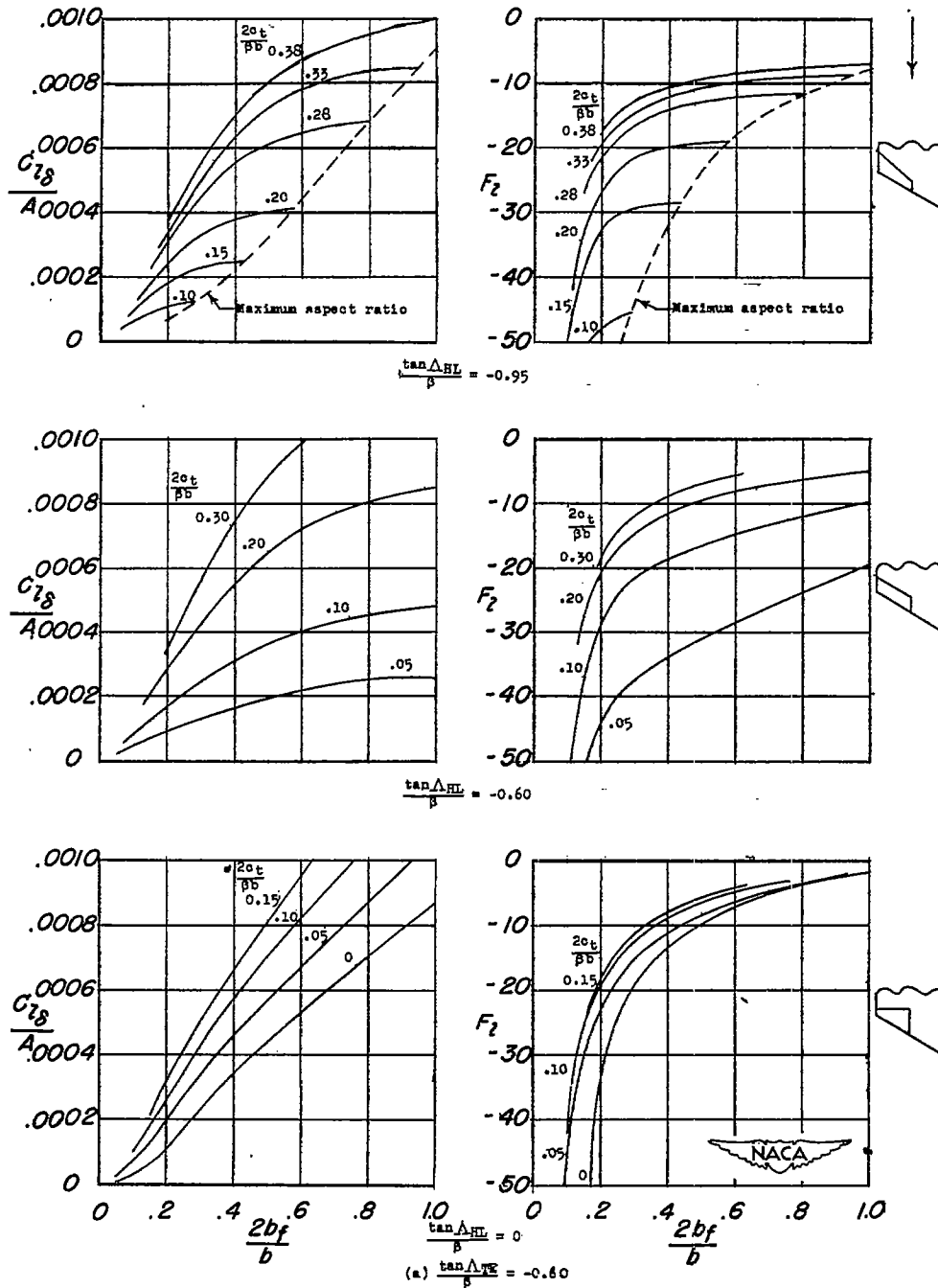
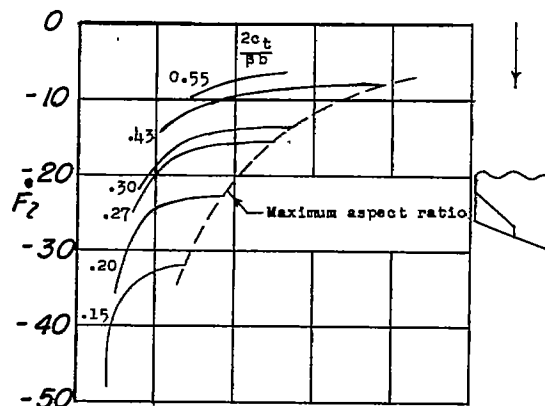
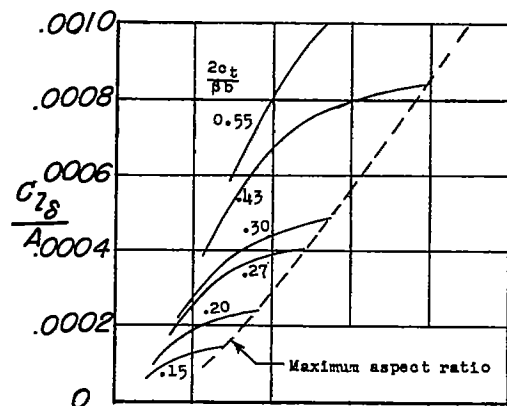
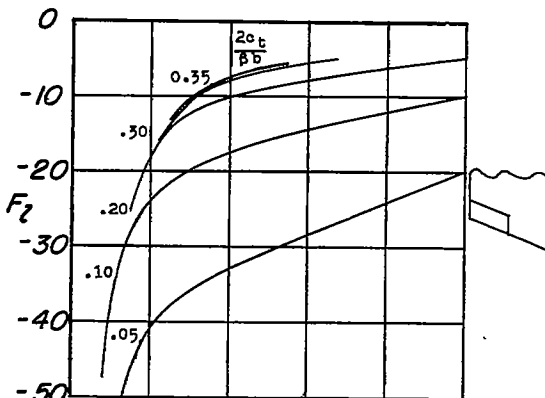
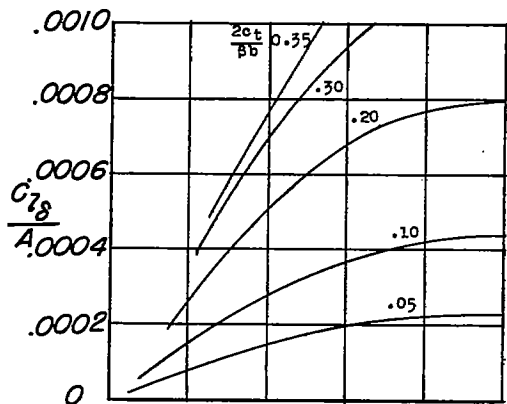


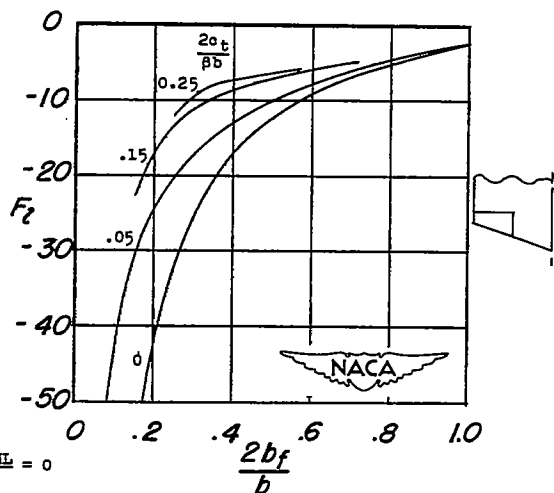
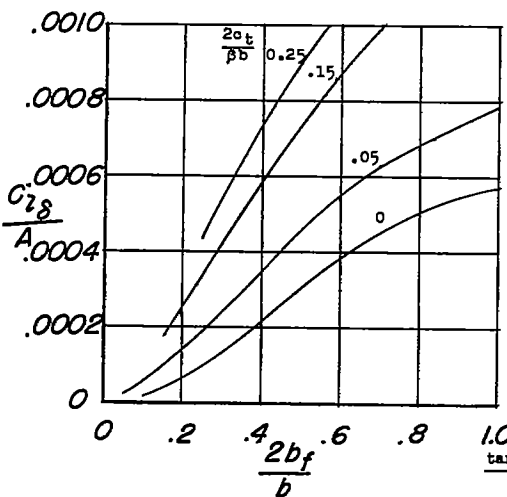
Figure 6.- Variations with shape and size of the values of  $\frac{C_{L\delta}}{A}$  and  $F_L$  for controls located at wing tips.  $\left( F_L = \beta \cos \Delta_{HL} \frac{L'}{H} = \frac{\delta_1}{57.3} \frac{\beta L'}{2W} \right)$



$$\frac{\tan \Delta_{HL}}{\beta} = -0.95$$



$$\frac{\tan \Delta_{HL}}{\beta} = -0.40$$



$$(b) \frac{\tan \Delta_{TE}}{\beta} = -0.40$$

Figure 6.- Continued.

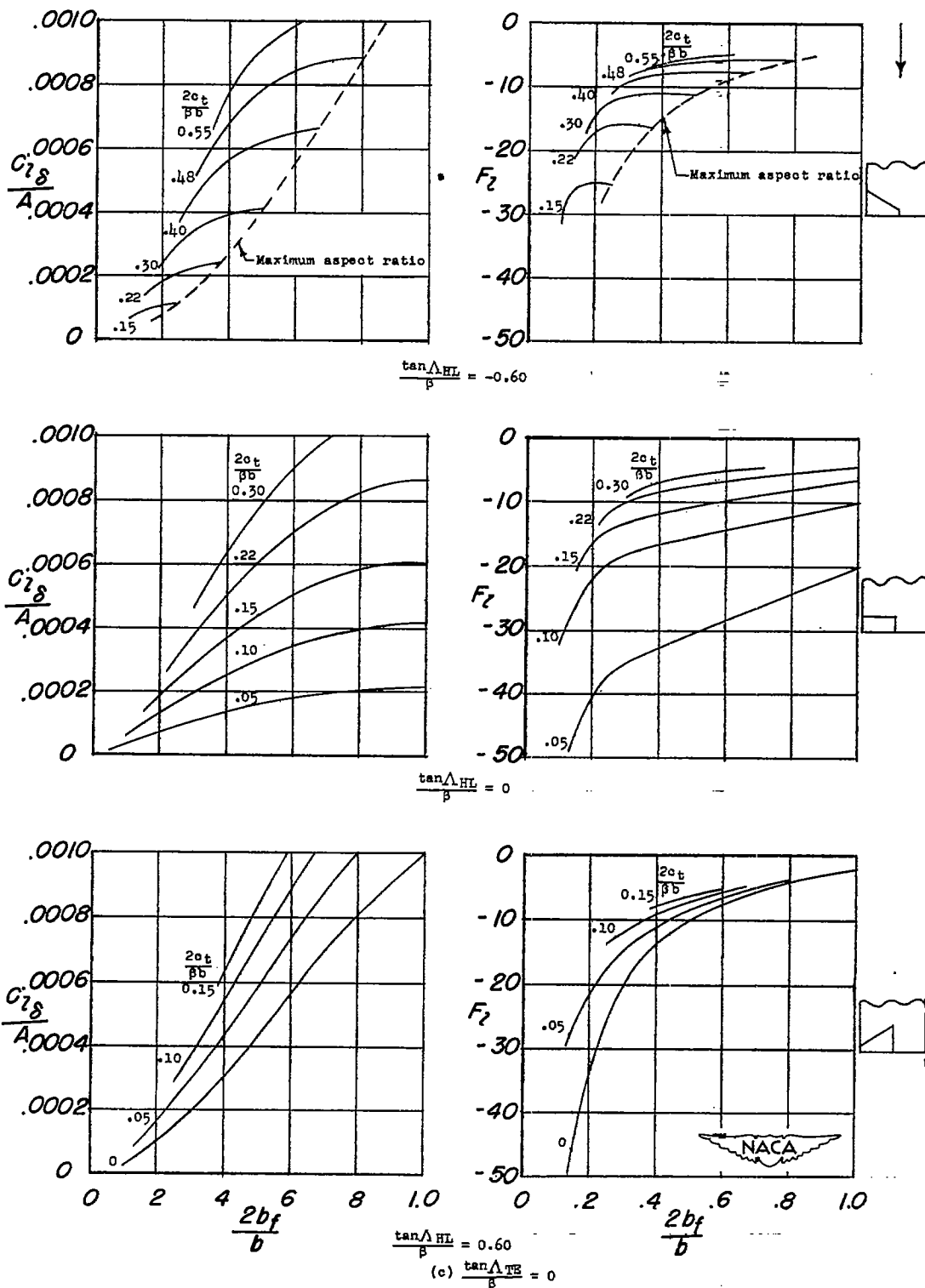


Figure 6.- Continued.

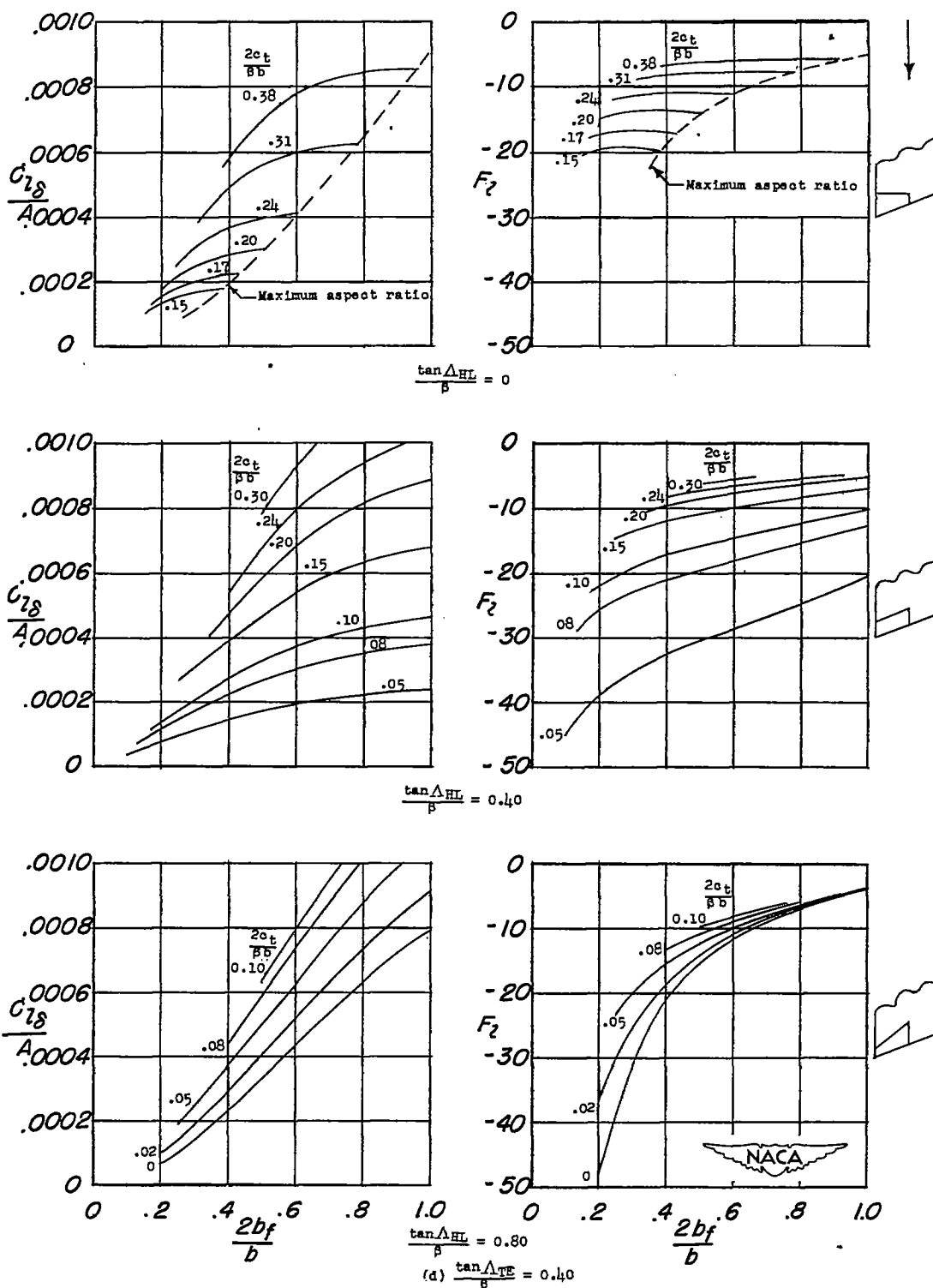


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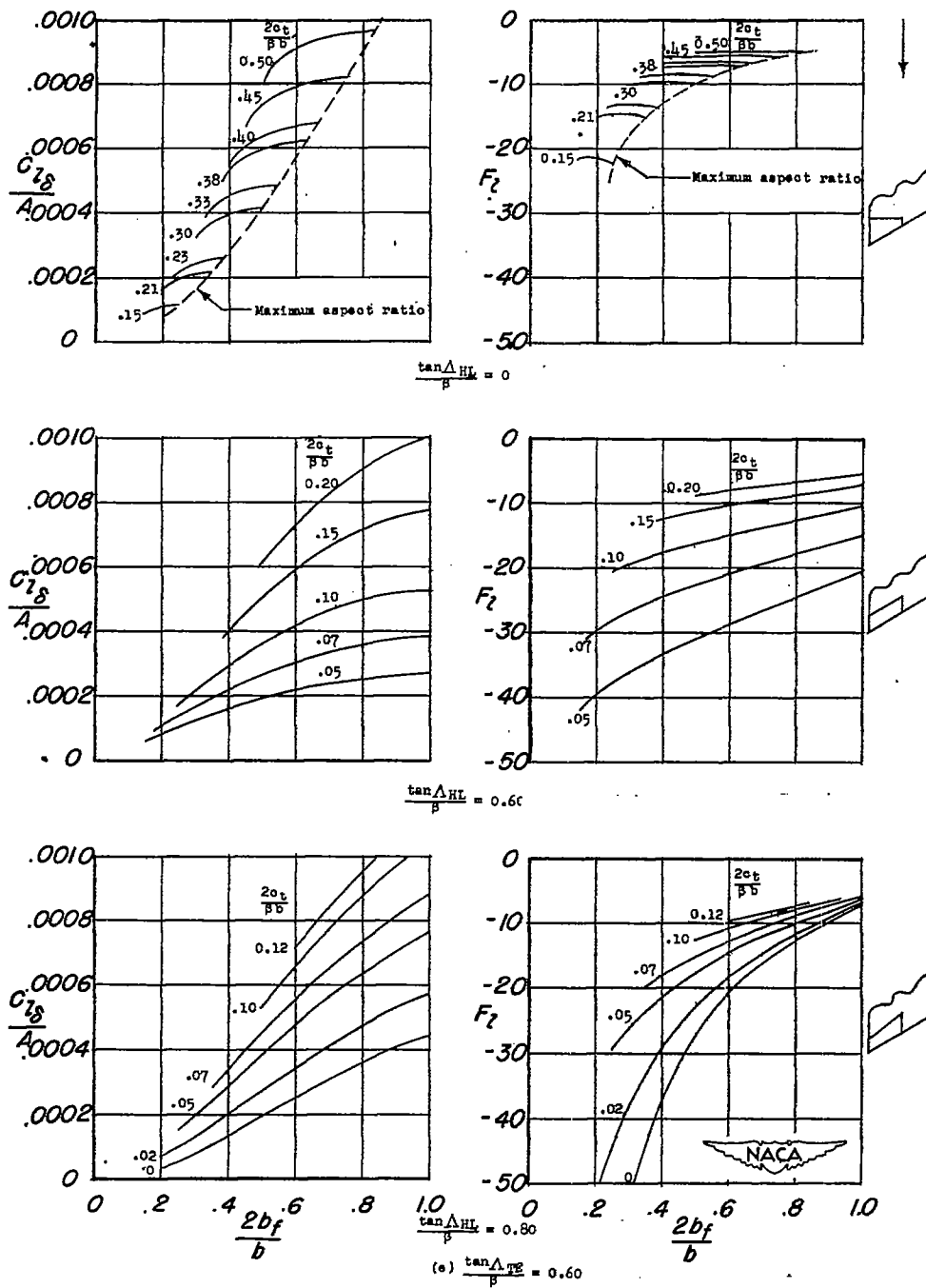


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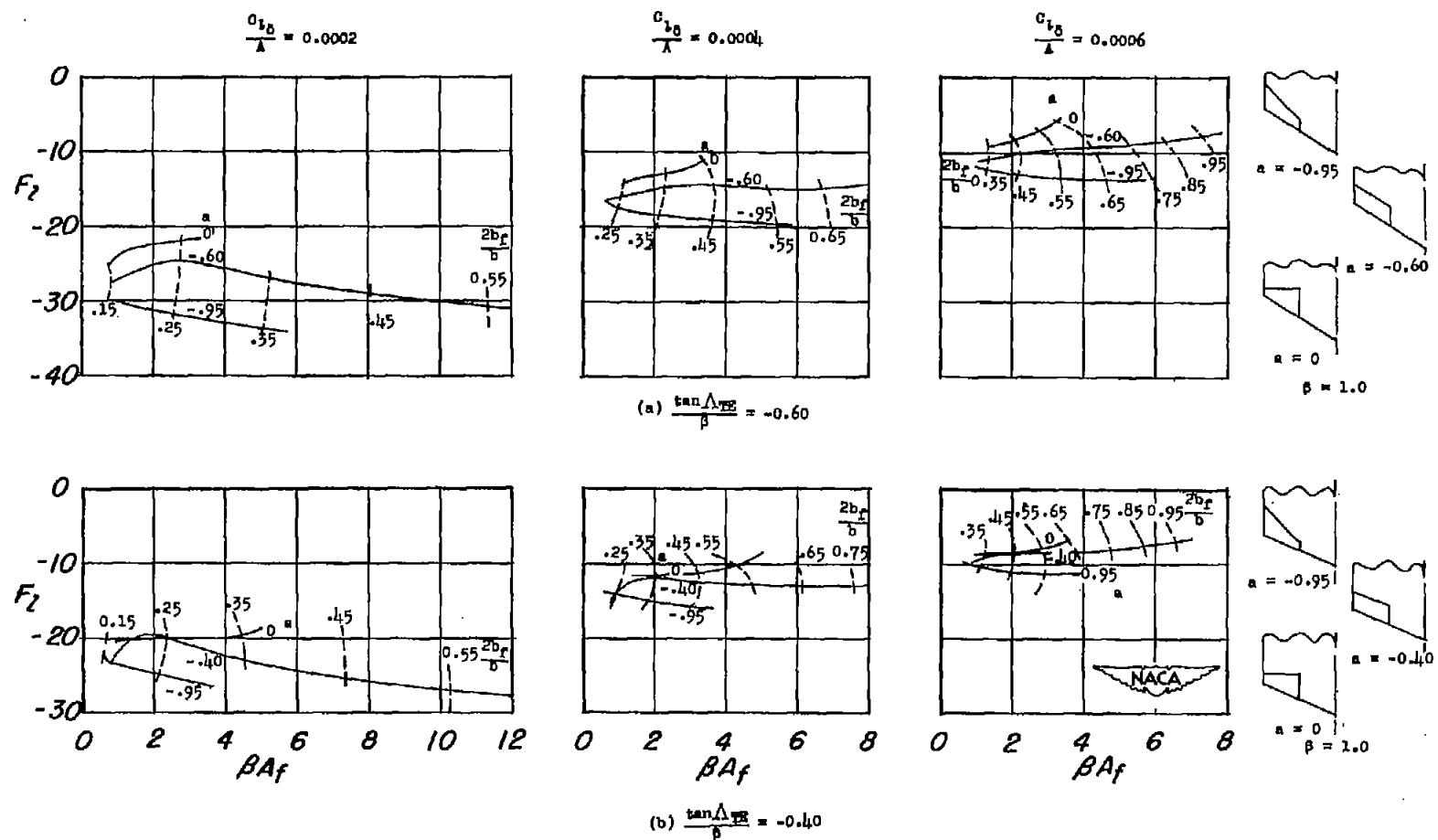


Figure 7.- Variations with plan form of the values of  $F_L$  for controls located at wing tips and having values of  $\frac{Cl_{\delta}}{A} = 0.0002, 0.0004, \text{ and } 0.0006$ .

$$\left( F_L = \beta \cos \Lambda_{HL} \frac{L'}{H} = \frac{\delta_1}{57.3} \frac{\beta}{2} \frac{L'}{W} \right)$$

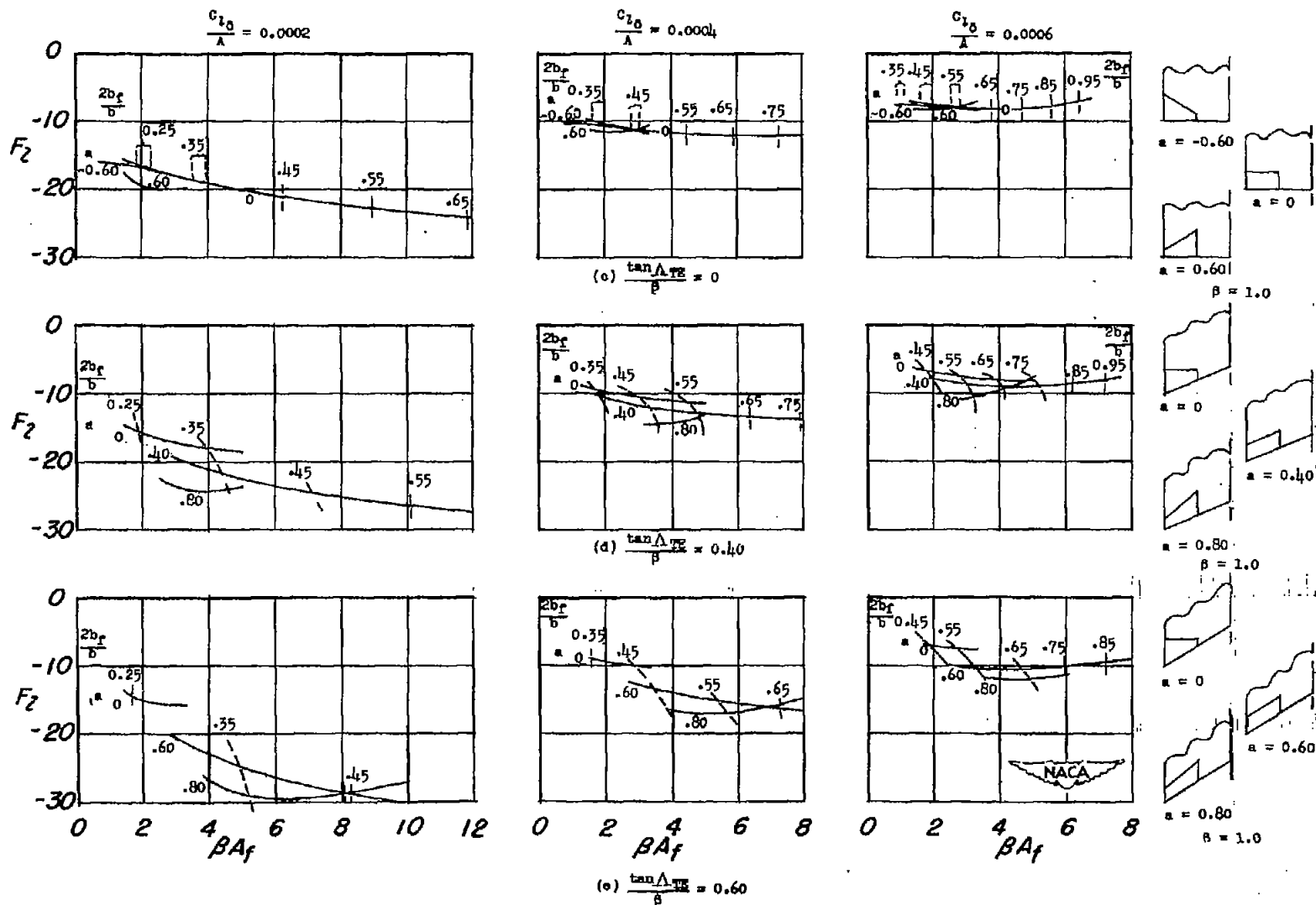


Figure 7.- Concluded.



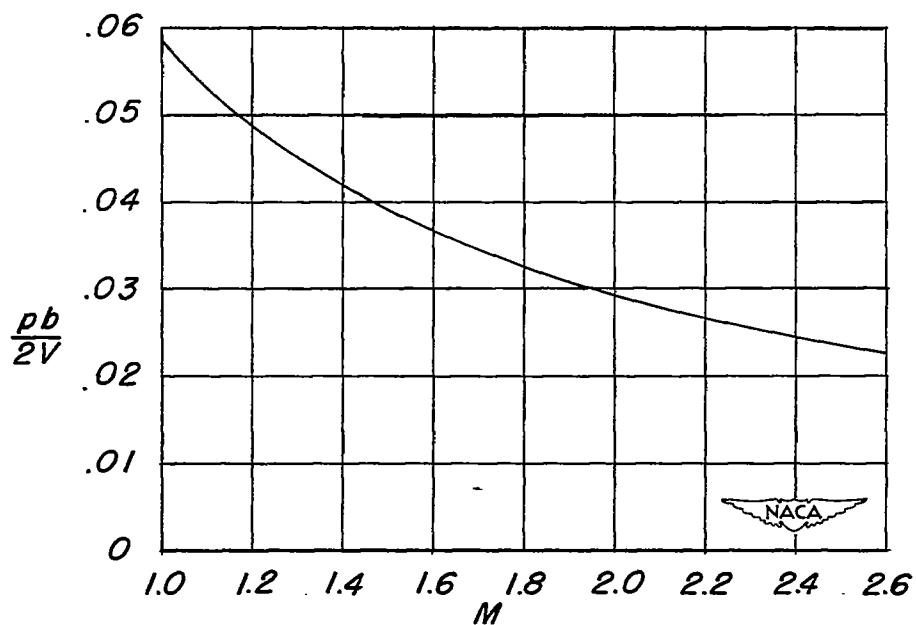


Figure 8.- Wing-tip helix angles corresponding to rolling rates of 3 radians per second for wings having spans of 38 feet and operating at an altitude of 40,000 feet.

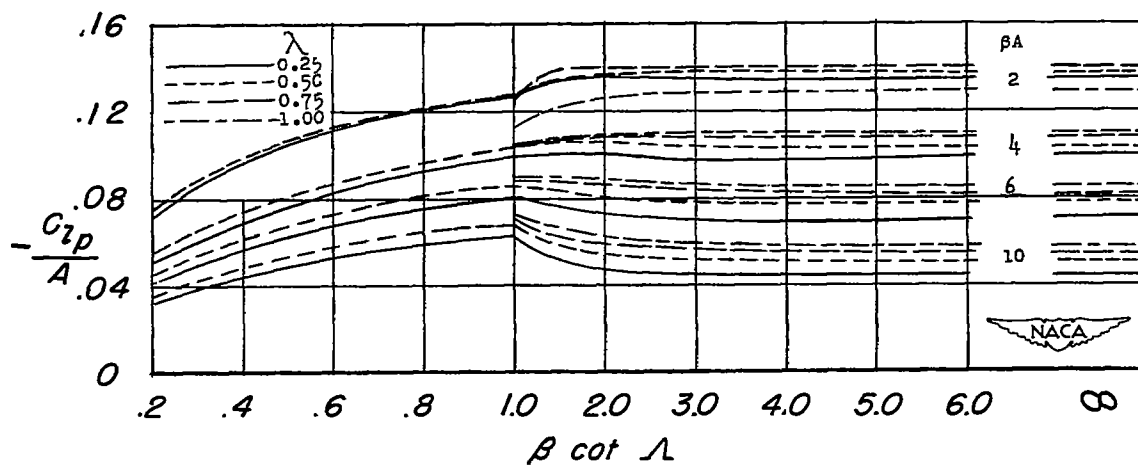


Figure 9.- Theoretical damping-moment coefficients for a range of wing plan forms and Mach numbers.

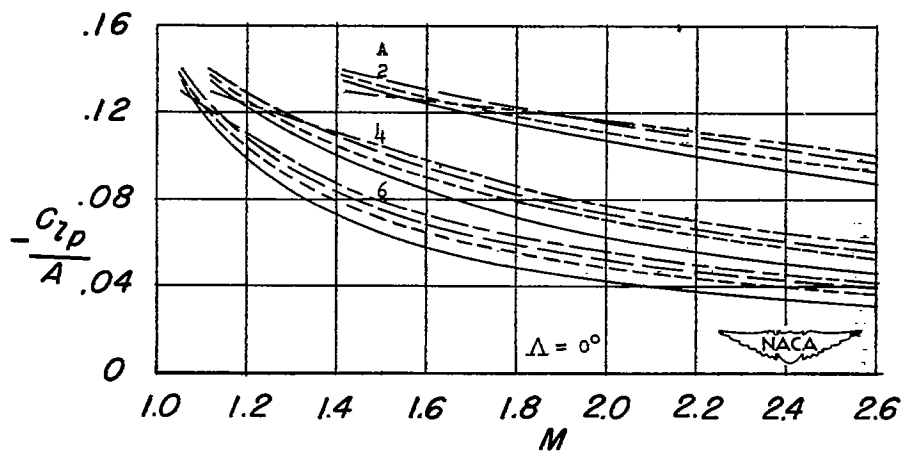
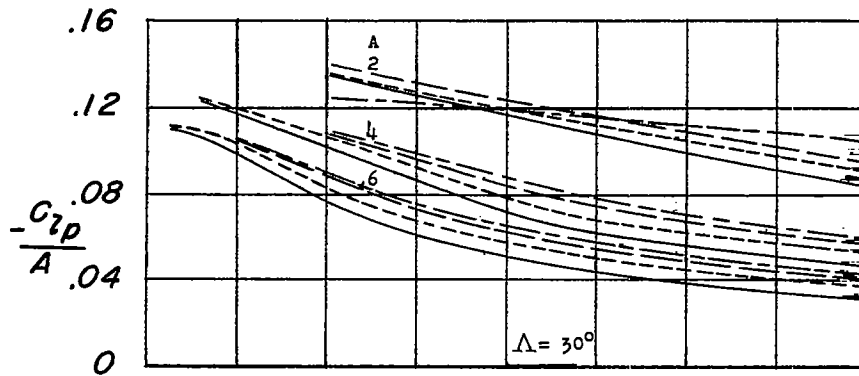
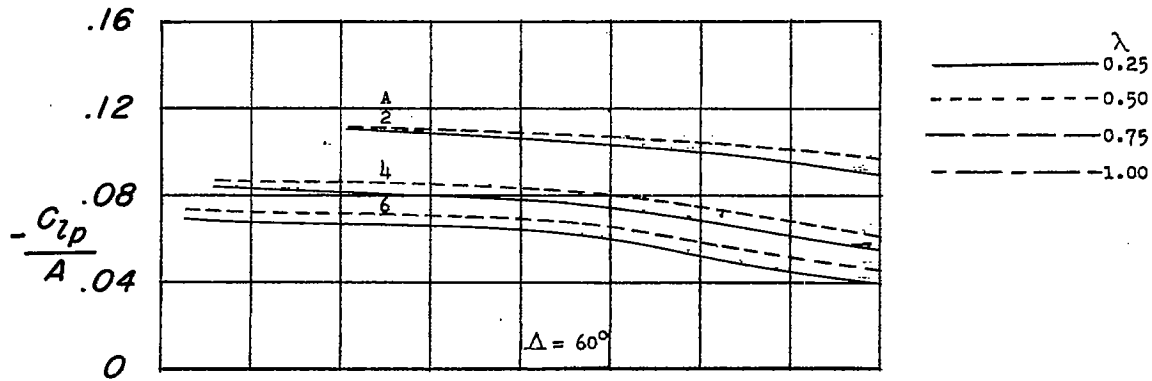
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Figure 10.- Some illustrative variations of wing damping-moment coefficients with Mach number.

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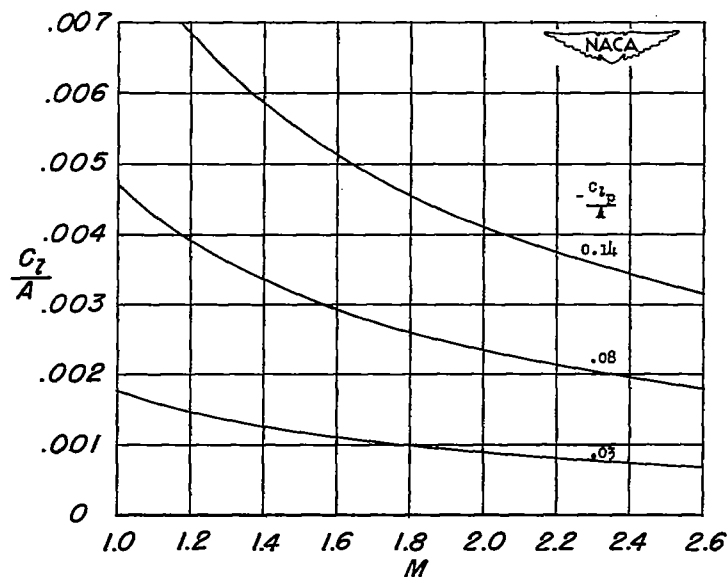


Figure 11.- Rolling-moment coefficients required to produce rolling rates of 3 radians per second for wings having spans of 38 feet and operating at an altitude of 40,000 feet.

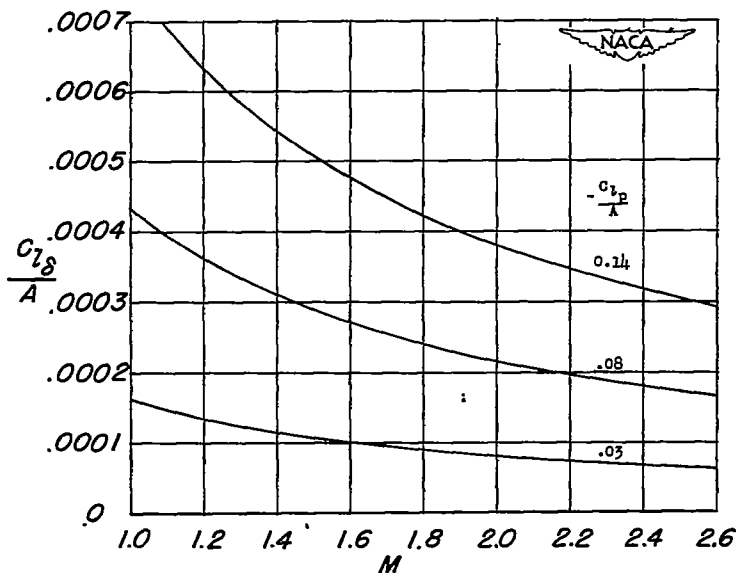


Figure 12.- Estimated theoretical rolling-moment-coefficient slopes corresponding to rolling-moment coefficients presented in figure 11 considering practical effectiveness of control  $\left(\frac{C_{l_\delta}}{A}\right)$  to be 36 percent of theoretical.

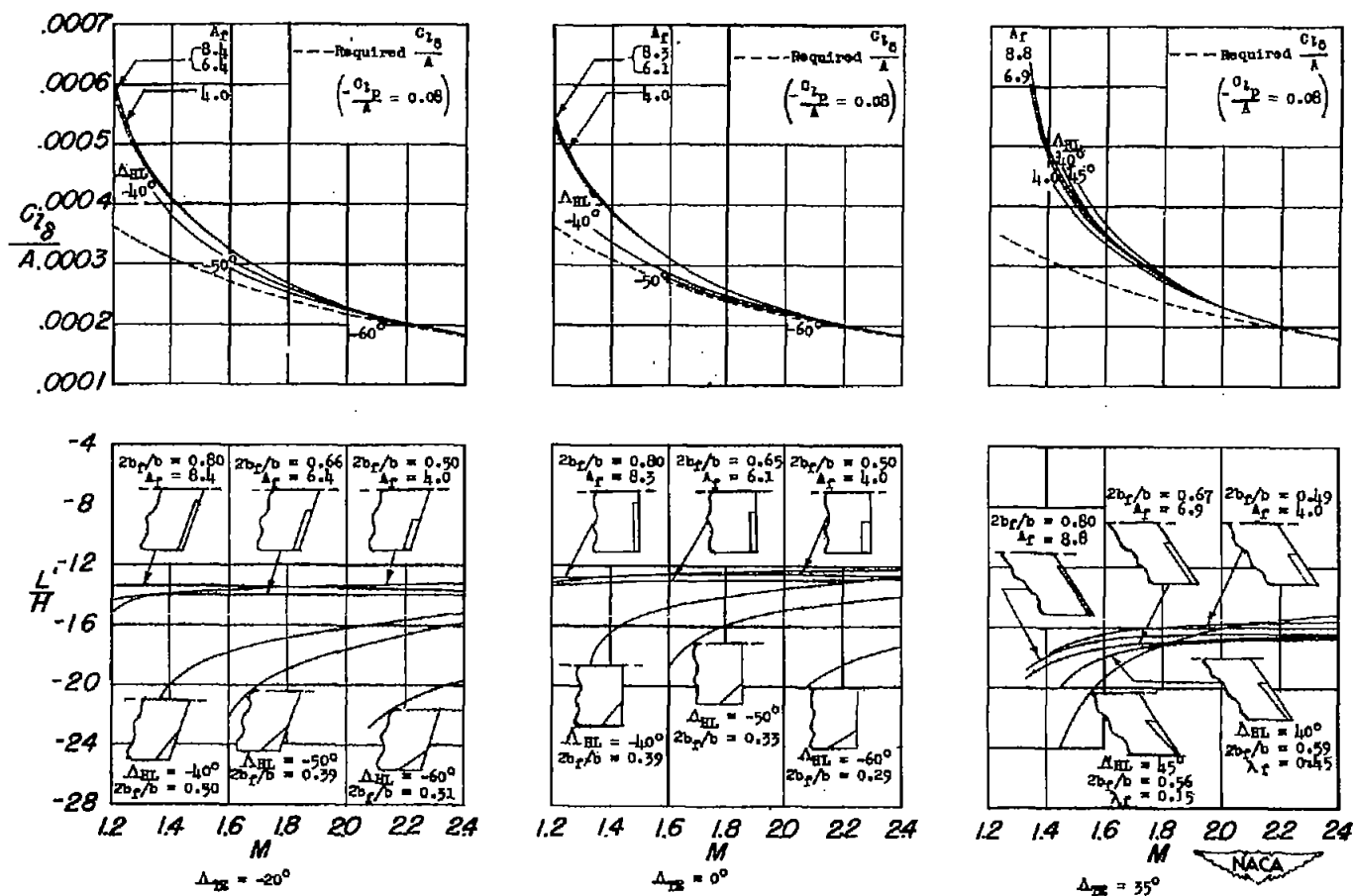


Figure 13.- Rolling-moment and hinge-moment parameters of various control plan forms on example wings having sweptforward, unswept, and sweptback

trailing edges and having damping coefficients of  $-\frac{C_{l_D}}{A} = 0.08$ .

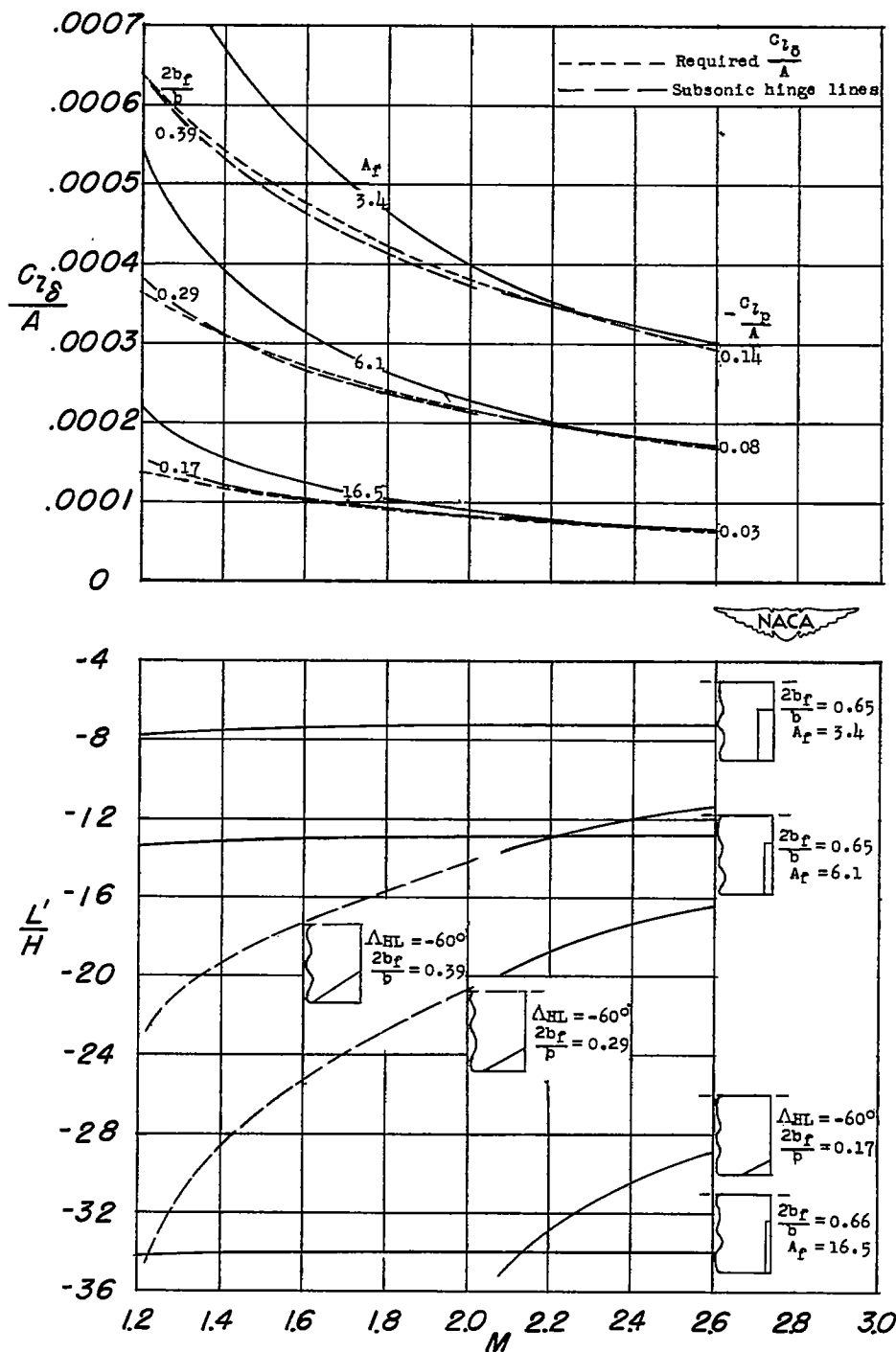


Figure 14.- Rolling-moment and hinge-moment parameters of various control plan forms on wings having unswept trailing edges and having damping

coefficients  $-\frac{C_{lp}}{A} = 0.03, 0.08, \text{ and } 0.14.$

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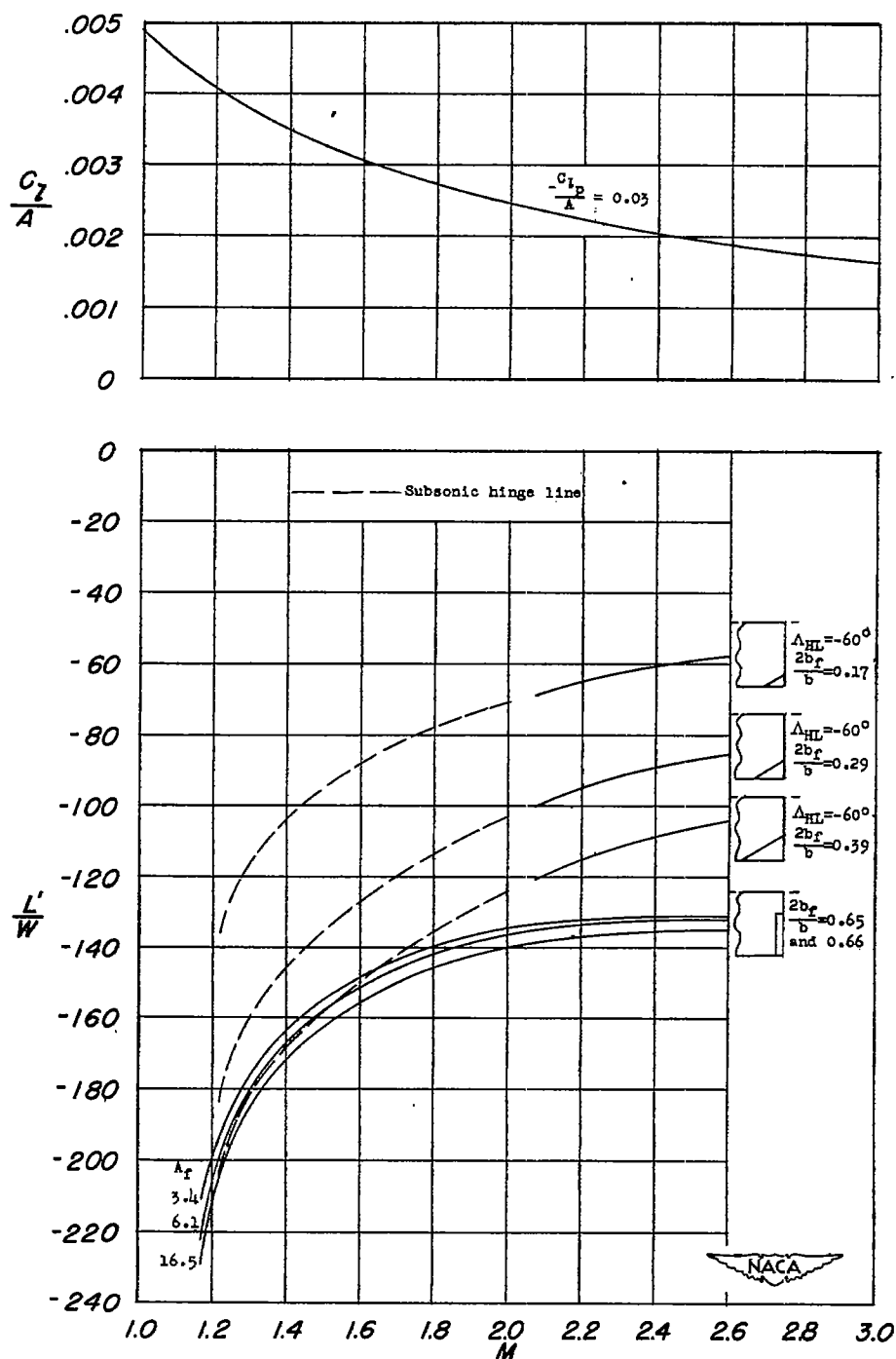


Figure 15.- Required rolling moments and ratios of rolling moment to deflection work for various control plan forms on an example wing having an unswept trailing edge and a damping coefficient  $\frac{C_{l_p}}{A}$  of 0.03.

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